Hysteresis Phenomena in the Response of the Oregonator under the Influence of an External Periodic Force

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ABSTRACT

Many systems expressed by nonlinear differential equations exhibit hysteresis phenomena. We have found the hysteresis of the Oregonator responding to an external periodic force. Taking the amplitude of the external periodic force as a control parameter and the frequency as a constant, we found by changing the control parameter with time. The oscillation modes show a hysteresis phenomenon.

1. INTRODUCTION

A hysteresis phenomenon is observed in a system expressed by a nonlinear differential equation or a set of nonlinear differential equations when exerted by an external force[1]. Famous examples are Van der Pol's and Duffing's equations[1]. We have observed the hysteresis on the Oregonator responding to an external periodic force.

The Oregonator which simulates the Belouzov-Zhabotinskii (B-Z) reaction[2] under the influence of an external periodic force is expressed by a set of nonlinear differential equations as follows[3-5],

$$\varepsilon\frac{d\xi}{d\tau}=\xi+\eta-q\xi^2-\xi\eta,$$

$$\frac{d\eta}{d\tau}=-\eta-2h\rho-\xi\eta,$$

$$\frac{d\rho}{d\tau}=(\xi-\rho)/p+A_z\cos\omega\tau, \qquad (1)$$

where ξ, η, ρ are the concentration of $HBrO_2$, Br^-, Ce^{4+}, respectively, p, q, h, ε are parameters and are taken as fixed constants: 2.0, 0.006, 0.75, 0.03, respectively, and $A_z\cos\omega\tau$ is the external periodic force. We use the same notation convention as that used by Tyson[5].

In previous papers[6),7)], we have studied the response of the Oregonator under an external periodic force $A_z\cos\omega\tau$. We have observed the oscillation modes of the second component η in case when ω is varied but $A_z=-40$ being a fixed constant. The appearance of m-peak oscillations (m-P.O.) : π (m) and a double periodic oscillation: $\pi_{p,q}$ (m,m+1) consisting of p, m-P.O.'s and q, m+1-P.O.'s with changing the control parameter ω, have been reported[6].

In this paper we take A_z as a control parameter and varying with time while ω as being a fixed constant: 3.14159. By changing A_z with time, we have observed a hysteresis phenomenon in the response in periodic oscillation modes of the second component η. When the value of A_z is being varied with time, a transition of the oscillation mode takes place at a certain values of A_z. The transition point differs in the case of increasing or decreasing of A_z with time. A hysteresis phenomenon is confirmed by the study described in the following sections.

2. RESULTS ON HYSTERESIS PHENOMENA

When A_z decreases linearly in time from 40 to 20, the oscillation mode of η changes from π (2) to π (3) at $A_z=31.6$ as is shown in Fig.1.

On the other hand, when A_z is increased from 20 to 40, a 3-P.O. mode changes to a 2-P.O. mode at a transition point $A_z=32.7$. The transition point differs in the case when A_z is decreasing or increasing. This difference is shown in Fig.2, where the —— solid lines (or······dotted lines) represent the minimum value of η when the value of A_z is decreasing (or increasing) in time, while ω is being fixed at 3.14159. The rate of change of A_z with time: $dA_z/d\tau$ is taken as 0.04.

In the region (I): $A_z>32.7$, the oscillation mode is a 2-P.O. mode and in the region (II): $A_z<31.6$, it is a 3-P.O. mode. In the two regions (I) and (II), the oscillation modes are independent of whether A_z is increasing or decreasing. But in the region (III): $31.6<A_z<32.7$ (we call the region a transition region) when A_z

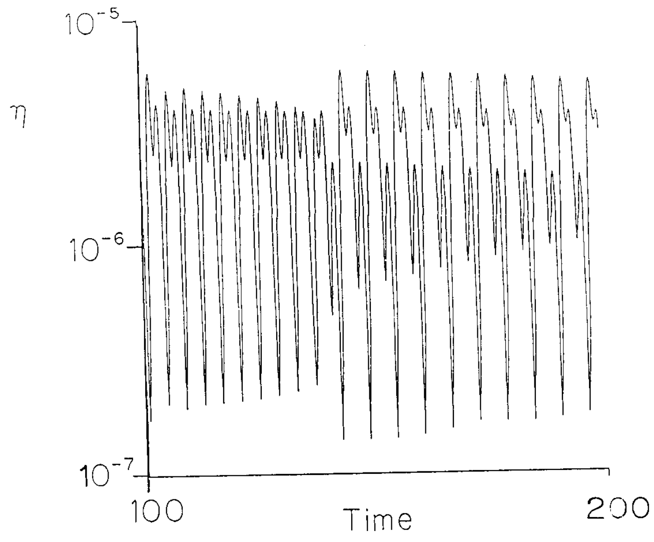

Fig.1. The change of the oscillation mode of the Oregonator in the component η from a 2-P.O. mode to a 3-P.O. mode when the amplitude of the external periodic force: $A_z \cos \omega\tau$, is decreased linearly with time from $A_z=40$ to 20 while the frequency ω being fixed at 3.14159.

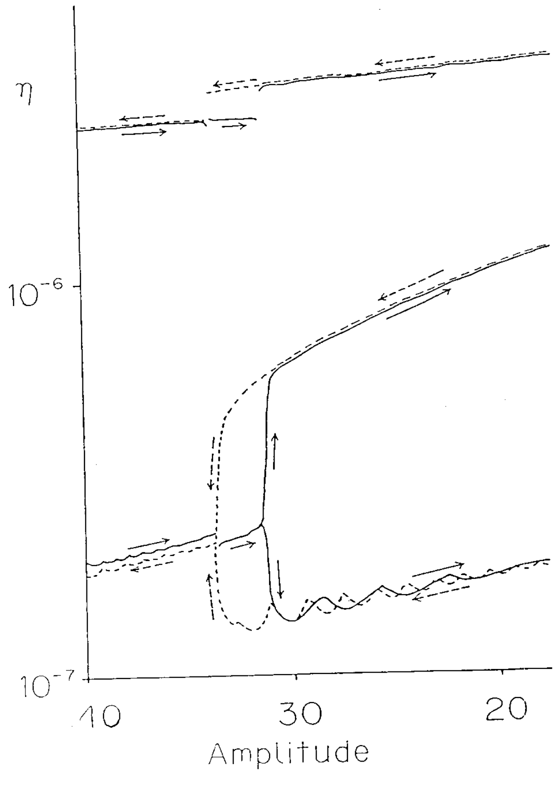

Fig.2 The minimum points of the temporal oscillation of the η-component when the amplitude of the external periodic force is increased from $A_z=20$ to 40 linearly in time (indicated by·········) and then is decreased to $A_z=20$ (indicated by———) while the frequency is being fixed constant: 3.14159. In the region : $32.7>A_z>31.6$ The trajectories show a typical hysteresis phenomenon.

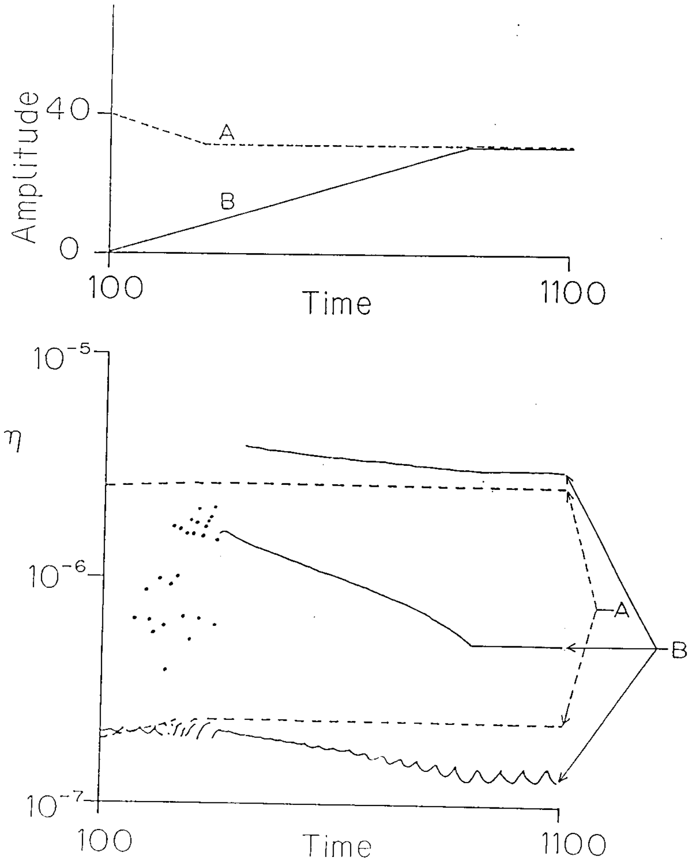

Fig.3 i) The amplitude A_z of the external periodic force $A_z \cos \omega\tau$ is being varied with time, $\omega = 3.14159$.

ii) The response in the oscillation mode in the final state where $A_z = 32$. The oscillation modes are different for the two cases A (indicated by·········) and B (indicated by———) for the same value of $A_z = 32$.

is increasing, the oscillation mode is a 3-P.O. mode, while the oscillation mode is a 2-P-O. mode, when A_z is decreasing. Therefore, in the region (III) for the same value of A_z, the oscillation mode is different. The transition point: from a 2-P. O. mode to a 3-P.O. mode and a 3-P.O. mode to a 2-P.O. mode are different and depends on whether A_z is increasing or decreasing.

That the difference in the oscillation mode is not due to the delay effect in the response of the Oregonator to the change of the external periodic force is shown below. Fig.3 shows that in the response of the Oregonator, the delay effect does not exist.

Fig.3 shows that the value of A_z is changed from 0 to 32 in region (III). If the delay effect exists in the response of the Oregonator, the oscillation mode should change from a 3-P.O. mode to a 2-P.O. mode. But the oscillation mode does not

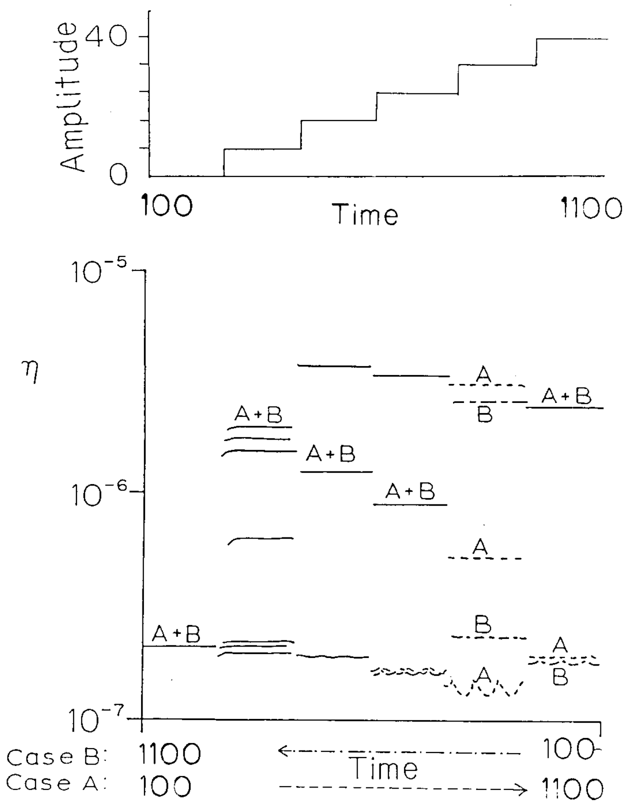

Fig.4 i) The value of A_z is increased (the case is denoted by A) or is decreased (the case is donoted by B) stepwise in a step of $A_z=8.0$.

ii) The oscillation modes agree in all the cases except in the transition region at $A_z=32$ where the mode is, for the case A): a 3-P.O. mode (indicated by— · — · — ·) but for the case B): a 2-P.O. mode (indicated by ———) and for the cases A) and B) agree (indicated by——).

change from a 3-P.O. mode for all time and no transition from a 3-P.O. mode to a 2-P.O. mode takes place. On the other hand, when A_z is decreased from $A_z=$ 40 to 32, the oscillation mode remains in a 2-P.O. mode for all time. Therefore, the difference in the oscillation mode for a value of $A_z=32$ is due to the hysteresis of the Oregonator and is not due to the delay effect of the response of the Oregonator.

To see more in detail the characteristic feature of the phenomenon we increased or decreased A_z in stepwise.

Case A: increase A_z successively in stepwise: $A_z=0,8,16,24,32,40$ (in each step increase of $A_z=8$) and case B: decrease A_z stepwise : $A_z=40, 32, 24, 16, 8, 0$. The result is illustrated in Fig.4. Fig.4. shows that the oscillation modes for the two cases agree for the five steps excepting $A_z=32$. In the case of $A_z=32$, the

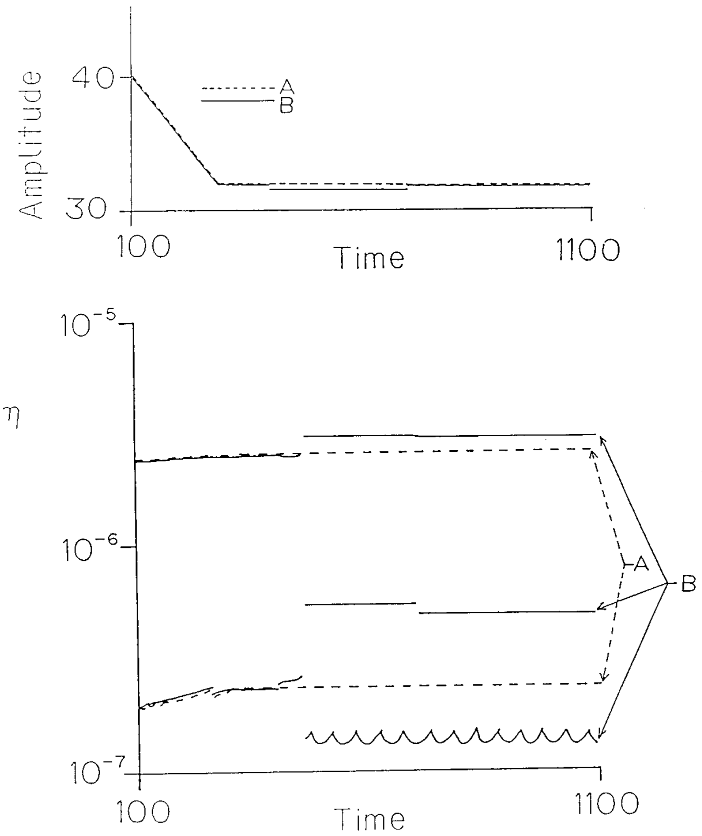

Fig.5. i) For the two cases the variation of the control parameter A_z with time are shown. Case A: the value of A_z is decreased from 40 to 32 (indicated by————). Case B: the value of A_z is decreased from 40 to 32 and maintained the value for a while and is then decreased 31.6. After maintainning for a while, bring the value of A_z back again to 32 (indicated by——).

ii) The responding oscillation modes are different for the two cases though the final values A_z are the same.

oscillation mode in 3-P.O. mode in case A and in a 2-P.O. mode in case B. Thus, for the two cases, the oscillation modes are different and remain different to time. Thus, there is no delay effect. The Oregonator responds immediately to the stepwise change of A_z.

Another example is illustrated in Fig.5. Fig.5 shows the response in η when the value of A_z is decreasing from 40 to 32 and then maintaining the same value for the rest of time. Then, the oscillation mode remains in a 2-P.O. mode all time. On the other hand, when the value of A_z is decreased from 40 to 32 and maintained the value for a while and then is decreased to 31.6. Then, the oscillation mode changes from a 2-P.O. mode to a 3-P.O. mode. After a while, bring the value of A_z back again to 32. the oscillation mode does not go back to a 2-P.O. mode but remains in a 3-P.O. mode. So that in the two cases, the oscillation modes are not the same for the same value of A_z. The difference depends on the variation history of A_z.

3. CONCLUSIONS

When the value of A_z is varied with time, the transition point of an oscillation mode is different according to whether A_z is increasing or decreasing. This difference is not due to the delay effect but is due to a hysteresis phenomenon. In the region $31.6 < A_z < 32.7$, the oscillation mode is a 2-P.O. mode or a 3-P.O. mode according as A_z is increasing or decreasing and depends on the time variation history of A_z. The hysteresis phenomenon is observed when the value of A_z is increased or decreased in stepwise. Therefore, the Oregonator exhibits hysteresis due to the external periodic force applied on the third component ρ in eq. (1).

It is interesting to study the hysteresis phenomena where an external periodic force is applied on the first and the second components and the frequency ω is changed with time while A_z is being fixed. But, it seems to be more complicated than we have reported here. The study is in progress and will be reported shortly elsewhere.

Many macroscopic properties of a piece of material exhibit phase transitions and hysteresis phenomena. But, in point of view of the critical exponents and in other characteristic features, whether the hysteresis phenomena of a set of nonlinear differential equations under an external force which appear through the oscillation modes are the same kind as those of a phase transition of a meterial or not is not yet clear.

It is interesting, to obtain some sort of critical exponents is the hysteresis phenomena of the Oregonator at other bifurcation points, by the computer calculations and compare these characteristic features with those of the phase transition in the macroscopic properties of a material.

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