

Examples of Bohm-Aharonov Effect in Eigenvalue Problems

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Abstract

We discuss the Bohm-Aharonov effect through the eigenvalue problems of an electron in quantum mechanics. We discuss i) the eigenvalue of the z-component of the angular momentum and ii) the Landau's level of an electron. These examples show that the Bohm-Aharonov effect does really exist.

1. Introduction.

In the classical electromagnetic field theory, the electromagnetic field describes electromagnetism. However, in quantum theory, an electromagnetic field cannot completely describe all the electromagnetic effects exerted on an electron. Bohm and Aharonov showed that the electromagnetic potential is more essential than the electromagnetic field in quantum mechanics¹⁾.

Bohm and Aharonov's discussion can be lucidly explained through the concept of the nonintegrable phase factor²⁾ as follows: in a doubly connected region where the electromagnetic field is zero, the interference fringe shift of an electron depends on the phase factor

$$e^{i\alpha\varphi} \text{ for } \alpha \text{ noninteger constant, (1)}$$

where φ is the angular coordinate of the axial symmetric coordinate system. The phase factor (1) induces a non-zero vector potential, but the vector potential is curlless. Thus, the wave function of the electron is essentially affected by the vector potential and this effect is called Bohm-Aharonov effect.

In this paper, we shall show an effect by an example which is different from the ordinary examples, in the eigenvalue problems in quantum mechanics.

2. A case of the angular momentum eigenvalue.

The phase factor (1) is analogous to the z-component of the angular momentum part of the wave function. Thus, we may prove that the eigenvalue of the z-component of the angular momentum will shift by the phase factor (1). We take the axial symmetric coordinate system (r, φ, z) , and in doubly connected region where the electromagnetic field is zero, we take the phase simply to be $\alpha < 1$. By the phase factor (1), the z-component of the angular momentum operator of an electron \hat{l}_z will be transformed as follows:

$$e^{i\alpha\varphi} : \hat{l}_z = \frac{\hbar}{i} \cdot \frac{\partial}{\partial \varphi} \longrightarrow \hat{l}_z = \frac{\hbar}{i} \left(\frac{\partial}{\partial \varphi} - i\alpha \right), \quad (2)$$

since $e^{i\alpha\varphi} : \hat{p} \rightarrow e^{i\alpha\varphi} \hat{p} e^{-i\alpha\varphi}$, where \hat{p} is the momentum operator of the electron. In order that the wave function to be single valued, its φ part is $e^{im\varphi}$, where m is an integer. This means that the wave function cannot be gauge-transformed by the phase factor (1). Therefore, the eigenvalue of the operator \hat{l}_z is⁽³⁾

$$\hbar(m - \alpha). \quad (3)$$

Thus, the phase factor (1) causes the shift in the z-component of the angular momentum eigenvalue by an amount α and this is the Bohm-Aharonov effect in an eigenvalue problem.

From the above conclusion, we considered the effect in the energy eigenvalue of an electron. However, in this case, we cannot observe the effect, since the energy eigenvalue in this system is a continuous one and difficult to ascertain the effect. To single out the effect we have to take up a discrete energy eigenvalue. Thus, we shall consider in the next section that an example of an appropriate external field is exerted on an electron is given to see the level shift in the discrete energy eigenvalue due to the Bohm-Aharonov effect.

3. A case of the energy eigenvalue—shift of Landau's levels.

First we consider an electron in a stationary magnetic field given in the

axial symmetric coordinate system, as

$$H_r = H_\varphi = 0, \quad H_z = H = \text{constant}. \quad (4)$$

Thus, the energy eigenvalue of the electron is given by Landau's level as follows⁴⁾:

$$E_n = \frac{\hbar |e| H}{Mc} \left(n + \frac{|m| + m + 1}{2} \right) + \frac{p_z^2}{2M}, \quad (5)$$

where e , M and p_z is the charge, mass and z-component of the momentum eigenvalue of the electron, respectively, n is zero or a positive integer and m is also an integer.

Now, since m is a quantum number of the z-component of the angular momentum of the electron, it is clear by (3) that the phase factor given by (1) changes to the following :

$$e^{i\alpha\varphi} : m \rightarrow m - \alpha, \quad (6)$$

where $\alpha < 1$. Therefore, from (6), we may immediately show that the Landau's level (5) shifts by the phase factor (1) as follows :

$$e^{i\alpha\varphi} : E_n \rightarrow E'_n = \frac{\hbar |e| H}{Mc} \left(n + \frac{|m - \alpha| + m - \alpha + 1}{2} \right) + \frac{p_z^2}{2M}. \quad (7)$$

Thus, from the expression (7), we may in principle observe the Bohm-Aharonov effect in an experiment of the energy spectrum in the quantum mechanical system.

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References

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