Quasi-one-dimensionally Tapered Planar Networks

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(Received September 20, 1983)

Abstract

Extrapolating the one-dimensional analysis of a transmission line to a two-dimensional physical network, it becomes clear that a coaxial multi-layers structure behaves as a nonuniform pseudo-distortionless transmission line. Such a curved multi-layers structure, however, is unsitable with respect to the manufacture. It thus is necessary to approximate a pseudo-distortionless transmission line by a planar multi-layers structure. A new graphical design described here is available for monotonously tapered planar networks with quasi-one-dimensional current-flow.

1. Introduction

Two-dimensional networks have attracted special interest recently. The circuit elements composing a two-dimensional network are much smaller than wavelength in one direction only, but are comparable to wavelength in the other orthogonal directions. One of the two-dimensional physical networks is a multi-layers structure composed of isotropic media with uniform thickness. When such a multi-layers structure is excited by the wavelength far longer than the thickness of the interior dielectric medium, the propagating mode is TEM mode only.

2. Coaxial Multi-layers Structure

Tapered pseudo-distortionless transmission lines have been analized using assumption of one-dimensional current-flow. The differential equation describing the voltage in a pseudo-distortionless transmission line may be written as

$$\frac{d^2}{dx^2} V(x)+\frac{d}{dx} lnf(x) \frac{d}{dx} V(x)-(R_0+j\omega L_0)(G_0+j\omega C_0)V(x)=0 \qquad (1)$$

where $V(x)$ is the phasor voltage at the position x on the line and ω is the radian frequency of the excitation, when the inductance, resistance, capacitance, and conductance per unit length at the position x are, respectively, given by

$$L(x)=L_0f(x), \tag{2}$$

$$R(x)=R_0f(x), \tag{3}$$

$$C(x)=C_0f^{-1}(x), \tag{4}$$

and
$$G(x)=G_0f^{-1}(x). \tag{5}$$

The diffusion equation describing the voltage in a multi-layers structure may be represented by

$$\nabla^2V(x, y)-(R+j\omega L)(G+j\omega C)V(x, y)=0 \tag{6}$$

where $V(x, y)$ is the phasor voltage at the position (x, y) on the surface of layers, ω is the radian frequency of the excitation, and L, R, C and G are, respectively, the inductance, resistance, capacitance and conductance per unit square.

A particular class of two-dimensional orthogonal coordinate system will may separate the diffusion equation (6) into two ordinary differential equations. In an effort to extrapolate the one-dimensional analysis of the pseudo-distortionless transmission line to a two-dimensional physical network, a new orthogonal coordinate system may be assumed. Let the assumed coordinate system be called the (ϕ, ψ) system. One set of the ϕ lines is assumed to lie along the equipotential lines in the two-dimensional network. Note the assumption is equivalent to assuming the existence of an orthogonal coordinate system in which the diffusion equation (6) may be separated into two ordinary differential equations, one of which has the same form as the one-dimensional differential equation (1).

Expressing the diffusion equation (6) in the new coordinates yields

$$\frac{1}{h^2_\phi}\frac{\partial^2V}{\partial\phi^2}+\frac{1}{h^2_\phi}\frac{\partial}{\partial\phi}ln\frac{h_\psi}{h_\phi}\frac{\partial V}{\partial\phi}-(R+j\omega L)(G+j\omega C)V=0 \tag{7}$$

where the h's denote the metrics of the new coordinate system. Equating Eq. (7) to Eq. (1), we obtain

$$h_\phi=1 \tag{8}$$

and
$$h_\psi=f^{-1}(\phi)g(\psi) \tag{9}$$

where $g(\psi)$ is an undefined function ψ alone. The orthogonal coordinate system with the deduced conditions, (8) and (9), may exist on a surface of revolution. It thus is clear that a coaxial multi-layers structure behaves as a nonuniform pseudo-distortionless transmission line.

3. Planar Multi-layers Structure

The coaxial multi-layers structures are unsuitable to the microwave integrated circuit, and the manufacture of them is also technically difficult, for the nonuniformly curved surface. If any coordinate system is introduced to the planar multi-layers structure, the diffusion equation (6) may not be separated to the ordinary differential equation describing a pseudo-distortionless transmission line with a few exceptions. It thus is necessary to approximate the terminal behavior of the pseudo-distortionless transmission line by a planar multi-layers structure. Such a planar network shall be called the quasi-one-dimensional planar network, because there is not the perfectly one-dimensional current-flow with a few exceptions.

To obtain the basis for a graphical design procedure of the quasi-one-dimensionally tapered planar networks, the following explanations are made. Eq. (8) is interpreted as specifying that the spacing between the coordinate lines, $\phi = n\Delta\phi$, is equal all over. From Eq. (9) the network width measured along a ϕ line shall be proportional to $f^{-1}(\phi)$, because the capacitance or area per unit length measured normal to the equipotential lines is specified to be proportional to $f^{-1}(\phi)$. Denoting the network width by the symbol $w\ (\phi)$, we obtain

$$w(\phi) = \alpha f^{-1}(\phi) \tag{10}$$

where α is a positive coefficient.

The graphical design mentioned here is available for the quasi-one-dimensionally tapered planar network in which the network-width function $w\ (\phi)$ increases monotonously all over. The line $(\phi = 0)$ and the line $(\phi = l)$ represent, respectively, the high-impedance end and the low-impedance end. The network-width function $w(\phi)$ may be expanded as the sum of three positive terms:

$$w(\phi) = w(0) + w'(0)\phi + \int_0^{\phi} w''(x)(\phi - x)dx \tag{11}$$

To approximate the curve of Eq. (11) by a polygonal line, the network length l is divided into n equal-length sections with length $\Delta\phi$.

$$n \cdot \Delta\phi = l \tag{12}$$

The approximation of Eq. (11) may written as

$$w(\phi) = (w((\lambda-1)\Delta\phi) + (w(\lambda\Delta\phi) - w((\lambda-1)\Delta\phi))\phi/\Delta\phi)(u(\phi/\Delta\phi - (\lambda-1)) - u(\phi/\Delta\phi - \lambda) \tag{13}$$

where u denotes the unit step function and

$$\begin{aligned} w(\lambda\Delta\phi) = {} & w(0) + w'(0)\lambda\Delta\phi \\ & + ((w(\Delta\phi) - w(0))\lambda - w'(0)\lambda\Delta\phi) \\ & + \sum_{\kappa=1}^{\lambda-1} ((w((\kappa+1)\Delta\phi) - w(\kappa\Delta\phi)) - (w(\kappa\Delta\phi) - w((\kappa-1)\Delta\phi)))(\lambda - \kappa) \end{aligned} \tag{14}$$

For a better approximation to the orthogonal coordinates, the first and second terms of the right side in Eq. (14) may be constructed as the sector bounded by two coaxial arcs and two radiuses. The remaining terms may be constructed as the symmetric areas in which each is an integration of the individual sectors with centers on the open edge.

The construction procedure used to obtain the geometrical shape of the planar network is as follows. After deciding on the network length l and the network-width function $w(\phi)$, the high-impedance end is drawn as the arc with length $w(0)$ and with radius $w(0)/w'(0)$. A series of arcs with radius $\Delta\phi$ and with centers on the high-impedance end are drawn. The line $(\phi=\Delta\phi)$ is drawn as the envelope of the constructed arcs and its length is set to be $w(\Delta\phi)$. At the open edge of the planar network, the line joining both the ends of the line $(\phi=0)$ and the line $(\phi=\Delta\phi)$ is normal to the line $(\phi=\Delta\phi)$, because it is the radius of the arc forming the last segment of the line. Hence, this procedure approximates the boundary condition that the equipotential lines are normal to the open edge. These process of drawing arcs to determine the direction of the constant ϕ lines and measuring the length of the ϕ lines to locate the open edge is repeated to establish the remaining section of the quasi-one-dimensionally tapered planar network.

4. Conclusion

We have constructed the quasi-one-dimensionally tapered planar network. It is apparent that this construction does not satisfy the conditions deduced under the assumption of the orthogonal coordinate system. If these conditions were satisfied, then the ϕ lines would be orthogonal to the line joining the midpoints of the ϕ lines on the half plane. This is negative. While it is not true that the specified conditions are satisfied all over the planar network, the conditions are closely approximated on the open edges, the high-impedance end, the center line and their adjacent areas. The small perturbations do not appear to influence significantly the terminal behavior, however, it is the subject for a future study to confirm experimentally the influence.

Acknowledgment

The author wishes to thank Dr. J. Ishii for many useful discussions and suggestions relating to the work.

Reference

J. Ishii and C. Nacacima, "Practical structures of nonuniform pseudo-distortionless transmission lines," Bulletin, Faculty of Sci. & Tech. Kinki Univ., vol. 9, pp. 143-147, 1974.