On the Fixed Points of Elliptic Elements of Finitely Generated Function Groups

Shigeyasu KAMIYA

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In the previous paper [1], we showed some properties of the fixed points of elliptic elements of B-groups.

In this paper we study the similar prorerties in the case of finitely generated function groups.

Before stating our theorem we shall explain notation.

Let $E(G)$ be the set of fixed points of elliptic elements in $\Lambda(G)$. Let $E_d(G)$ (resp. $E_e(G)$) be the subset of $E(G)$ consisting of the fixed points of elliptic elements in some degenerate factor subgroup (resp. some elementary factor subgroup) of G.

Our result is the following theorem.

THEOREM. *Let G be a finitely generated function group with an ivariant component Δ_0. Then the following four propositions hold:*

(1) $E(G)=E_d(G)\cup E_e(G)$.

(2) For any $z\in E(G)$, if z is fixed by an elliptic element with the period $r\geqq 3$, then z can not lie on the boundaries of components except Δ_0.

(3) Let H be some factor subgroup of G. For $z\in\Lambda(H)$, if z is fixed by an elliptic element with the period $r\geqq 3$ and a parabolic element, then H is an elementary group.

(4) Let E be an elliptic element in G and a, b its fixed points. If any factor subgroup is not finite, both a and b can not lie in Δ_0.

1. Let us begin with recalling some notation and definitions.

Let G be a kleinian group, $\Omega(G)$ the region of discontinuity, and $\Lambda(G)$ the limit set of G. The component of $\Omega(G)$ are called components of G. A component Δ_0 of G is called invariant if $V(\Delta_0)=\Delta_0$ for every $V\in G$. If Δ is a comonent of G other than the invariant component, then the subgroup G_Δ of G keeping Δ invariant is called a component subgroup of G.

A kleinian group with an invariant component is called a function group. Throughout this paper we denote a finitely generated function group by G. A subgroup H of G is called a factor subgroup if H is a maximal subgroup of G with the following properties: the invariant component of H, which contains the invariant component of G, is simply connected; H contains no accidental parabolic transformations; if $V \in G$ is parabolic and the fixed point of V lies in $\Lambda(H)$, then $V \in H$. If H is a factor subgroup of G, then G is elementary, degenerate, or quasifuchsian.

2. Now we are ready to prove our theorem.

PROOF OF (1). It is well-known that an elliptic element in G is an element of some faetor subgroup of G and that the set of component subgroups equals the set of quasifuchsian factor subgroups. Using the above fact, in the same manner as in the proof of (1) in [1], we can show that $E(G)=E_d(G) \cup E_e(G)$.

PROOF OF (2). Take z in $E(G)$. Let E be an elliptic element with a fixed point z and its period r. Let Δ be a component except Δ_0. Assume that z lies on the boundary $\partial\Delta$. Since G_Δ is a quasifuchsian group, the element E does not belong to G_Δ. Therefore $\Delta \neq E(\Delta) \neq E^2(\Delta)$, so $z \in \partial\Delta \cap \partial E(\Delta) \cap \partial E^2(\Delta)$. This contradicts Theorem 5 in [3]. Hence z can not lie on the boundaries of components except Δ_0.

PROOF OF (3). If z is fixed by an elliptic element E with the period $r(\geqq 3)$ and a parabolic element P_1, then there exists a parabolic element P_2 such that $P_2(z)=z$ and $P_1^m \neq P_2^n$ for any non-zero integers m and n (cf. [1]). Since H is a factor subgroup of G, P_1 and P_2 belong to H. Using Lemma 2 in [1], we see that H is not a B-group. Therefore H is an elementary group.

PROOF OF (4). Suppose that a and b lie in Δ_0. It is known that E belongs to some factor subgroup H of G. If H is a quasifuchsian group, one of its invariant components, say Δ, is a component of G and so either a or b lies in Δ. Hence both a and b can not be contained in Δ_0. Next we consider the case of the group H which is degenerate. In this case, either a or b, say a, lies in $\Lambda(H)$. (cf. [1]). Therefore a is contained in $\Lambda(G)$. When H is an elementary group with one limit point, we can show by the same way as the above that both a and b can not lie in Δ_0.

Thus our theorem is completely proved.

References

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Department of Mechanical Science,
Faculty of Science,
Okayama University of Science