

# Second Order Phase Transition of a System of Direct-Product Group Symmetry

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The thermodynamical conditions for a second order phase transitions accompanied by the changes of a direct-product group symmetry are given according to the Landau theory. The present theory explains successfully the phase transitions of the system possessing order parameters which correlate with each other.

Since the publication of the phenomenological theory of Landau [1] on the relations between the second order phase transition and a symmetry change, recently many workers [2] have been mainly interested in the transitions of systems having a coupling between different degrees of freedom.

In this report we apply the Landau theory to the second order phase transitions accompanied by the changes of the direct-product group symmetry of two groups  $A$  and  $B$  possessing the different characters; these have no common element except for the identity and any operator of  $A$  commutes with any operator of  $B$ . Then, the present theory can give a good explanation for the transitions of the system having order parameters which correlate with each other, for examples the liquid crystals [3] and the ferroelectrics [4] which are, however, the systems giving arise to the first order phase transitions.

A density function  $\rho$  specifying the internal variables can be decomposed into a sum of the basis functions belonging to the irreducible representations of the direct-product group  $A \times B$ , these functions are formed by the basis functions  $\phi_\mu^\alpha$  and  $\psi_\nu^\beta$  belonging to the irreducible representations of  $A$  and  $B$  respectively.

$$\rho = \rho_0 + \sum'_{\alpha\beta} \sum_{\mu\nu} C_{\mu\nu}^{\alpha\beta} \phi_\mu^\alpha \psi_\nu^\beta \equiv \rho_0 + \delta\rho \quad (1)$$

where the sum does not contain the function belonging to the identical representation of  $A \times B$ .

Near the transition point, a thermodynamic potential  $\phi$  can be expanded into by a power series of  $\delta\rho$  about  $\rho_0$ , as  $\delta\rho$  becomes zero continuously in the second order

phase transition. We must construct  $\Phi$  by the invariants under all operations of  $A \times B$ . On assuming the functions have already been determined, we consider the invariants for the coefficients  $C_\mu^\alpha$  and  $D_\nu^\beta$ .

$$\Phi = \Phi_0 + \sum'_\alpha A_1^\alpha \sum_\mu C_\mu^\alpha + \sum'_\beta A_2^\beta \sum_\nu D_\nu^\beta + \sum''_{\alpha,\beta} A_3^{\alpha\beta} \sum_{\mu,\nu} C_\mu^\alpha D_\nu^\beta, \quad (2)$$

where each sum does not contain the functions belonging to the identical representations of  $A$  and  $B$ , and  $A$ 's are depend on the temperature and the external forces. Hereafter we will consider only the single representation of  $A \times B$ .

When we introduce, according to the Landau theory, order parameters  $\sigma$  and  $\eta$  which are concerned with the symmetry change of  $A$  and with of  $B$  respectively, and take account of the third and fourth order terms of those,  $\Phi$  can be given by the equation ;

$$\Phi = \Phi_0 + A_1\sigma^2 + A_2\eta^2 + B_1\sigma^3 + B_2\eta^3 + A_3\sigma^2\eta^2 + C_1\sigma^4 + C_2\eta^4. \quad (3)$$

It is to be noted that  $\sigma$  and  $\eta$  interact with each other through  $A_3$ . As the third order terms in equation complicate the problems, we will neglect them to see through the essence in our approach.

Under the constant external forces, the equilibrium values of  $\sigma$  and  $\eta$  are obtained by the roots of the first order partial differentiations  $\Phi_\sigma$  and  $\Phi_\eta$ ,

- (1)  $\sigma = 0, \eta = 0$  ;
- (2)  $\sigma = 0, \eta^2 = -A_2/2C_2$  ;  $\sigma^2 = -A_1/2C_1, \eta = 0$  ;
- (3)  $\sigma_l^2 = (A_2A_3 - 2A_1C_2)/(4C_1C_2 - A_3^2), \eta_l^2 = (A_1A_3 - 2A_2C_1)/(4C_1C_2 - A_3^2)$ .

These roots must be real and the minimum point of  $\Phi$ . The latter case is given by the conditions for the second order partial differentiations ;  $\Phi_{\sigma\sigma} > 0$  and  $\Phi_{\sigma\sigma}\Phi_{\eta\eta} - \Phi_{\sigma\eta}^2 > 0$ .

We study first case that when the temperature of the system is lowered down through the transition temperature  $T_1$ , the symmetry of the system changes from  $A \times B$  to  $A' \times B'$  where  $A'$  and  $B'$  are the subgroups of  $A$  and  $B$  respectively. In each temperature region, we have the equilibrium values of  $\sigma$  and  $\eta$  and the thermodynamical conditions ;

- (a)  $T \geq T_1$  ;  $\sigma=0, \eta=0, A_1, A_2 \geq 0$ ,
- (b)  $T < T_1$  ;  $\sigma = \pm\sigma_l, \eta = \pm\eta_l, A_3/2C_1 < A_2/A_1 < 2C_2/A_3$ .

The values of  $\sigma$  and  $\eta$  are continuous by the conditions  $A_1, A_2=0$  at  $T_1$ .

We study next case where the symmetry of the system changes from  $A \times B$  to  $A' \times B$  on passing through  $T_1$  and  $A' \times B$  to  $A' \times B'$  through  $T_2$  (assuming  $T_1 > T_2$ ). We get the followings under the conditions  $C_1, C_2 > 0$  ;

- (c)  $T \geq T_1$  ;  $\sigma=0, \eta=0, A_1 \geq 0, A_2 > 0$ ,

- (d)  $T_1 > T \geq T_2$ ;  $\sigma^2 = -A_1/2C_1$ ,  $\eta = 0$ ,  $A_1 < 0$ ,  $A_1 A_3 \leq 2A_2 C_1$ ,  
 (e)  $T < T_2$  ;  $\sigma = \pm \sigma_l$ ,  $\eta = \pm \eta_l$ ,  $4C_1 C_2 > A_3^2$ ,  $A_2 A_3 > 2A_1 C_2$ ,  $A_1 A_3 > 2A_2 C_1$ .

Our theory is more applicable to the systems giving rise to the first order phase transitions, in this case it can be easily shown that we must take account of the third order terms in eq. (3).

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### References

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