The Measurement of a Magnetic Field Strength in the Second Rotating Reference Frame

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To perform a rotary-saturation experiment, it is necessary to add an audio channel to a standard nuclear-magnetic resonance apparatus. In this paper, the method for measuring of the strength of the magnetic field produced by the audio coils is represented. Measurement is performed by treating with the behaviour of the magnetization in the second rotating reference frame. The error of the results by this method is at most a few percents.

I. Introduction

In 1955, Redfield[1] discovered the phenomena which he called rotary saturation in nuclear magnetic resonance. The technique based on this phenomena is of great practical utility for calibrating the strength of a radio-frequency magnetic field. In addition, the excellent point of this technique is that this enables us to observe multiple-quantum transitions in solids[2],[3]. To perform a rotary-saturation experiment, it is necessary to add an audio-freqency channel to a standard nuclear-resonance apparatus. Therefore, in order to analyze the results of this experiment, it is required to determine the strength of the audio-frequency magnetic field, $2H_a$, other than that of the radio-frequency magnetic field, $2H_1$. Usually H_1 is measured by 90° pulse method or the rotary-saturation method more acculately. On the other hand, it is difficult to measure H_a directly by these method.

In this paper, since we have found the method for measuring of H_a by considering the behaviour of the magnetization in the second rotating frame, we wish to report this method and the results. The things to be emphasized of our method are that one can directly measure the field strength without the knowledge for the geometrical parameters of the audio coils and the results obtained is not concern of the various parameters of the apparatus used. The apparatus used in our experiment is rotary-saturation apparatus which is consisted of the crossed-coil pulsed NMR apparatus with an audio channel, and has been described in our earlier paper²⁾. The audio field is applied along z direction in the laboratory frame. Resonant nuclei used in our experiment is proton nuclei in water doped with cupreous ions.

II. Experimental procedure and the method for measuring the strength of an audio-magnetic field

In this section, we shall explain the procedure for measuring the strength of an alternating magnetic field in order. The pulse sequence used in our experiment is shown in Fig. 1.

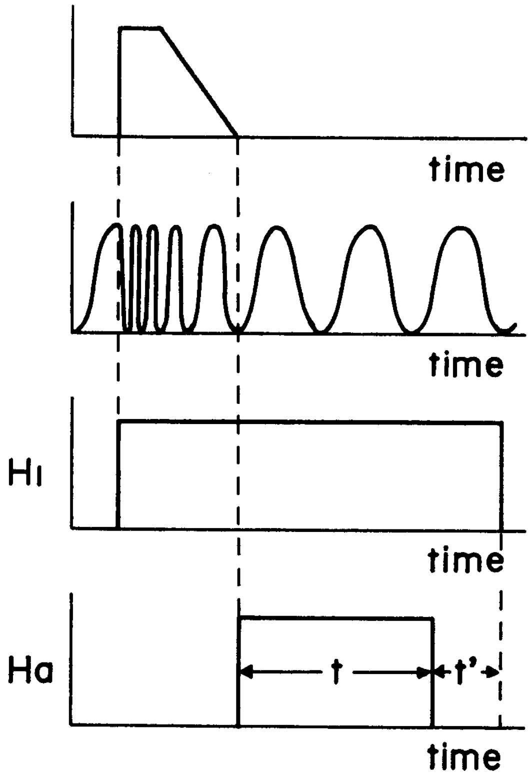

Fig. 1 The pulse sequence.

The spin system, consisting of the proton nuclei in water, is placed in a static magnetic field H_0 applied in the z-axis of the laboratory frame. After the thermal equiibrium is reached and a large magnetization developes, radio-frequency magnetic field of the frequency ω is pulsed on along the x-axis. Initial value of its frequency is far off resonance, and then adiabatically brought to the resonance value. At the end of this process, the magnetization M is aligned along the effective magnetic field H_e in the frame rotating at frequency ω about z-axis of laboratory frame. (This frame is so-called the first rotating frame.) After the Zeeman and spin-spin systems have had time to re-equilibrate in this frame, an audio-magnetic field of frequency ω_a is pulsed on along the z-axis of the laboratory frame.

In order to investigate the effect of H_a on the spin system, it is convinient to introduce the concept of the second rotating frame. This frame is defined as the frame rotating at frequency ω_a about H_e. In this frame, the effective mangetic field H_e' is static and the magnetization precesses about it. Then H_a is pulsed off, and after t_2 second H_1 is pulsed off. The remaining magnetization precesses about the static field H_0 in the laboratory frame and is detected before it decays appreciably. Fig. 2 shows the behaviour of the magnetization and the relation between the magnetic

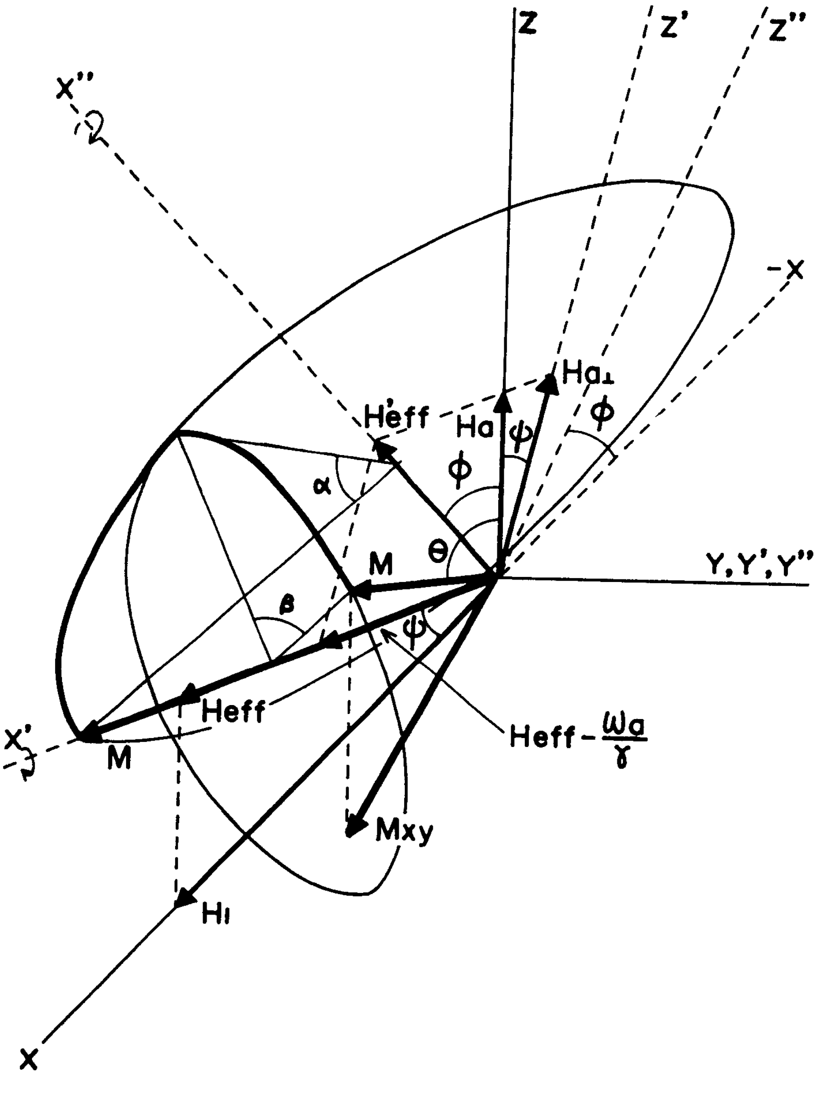

Fig. 2 The first and second rotating frames. (Symbols defined in text.)

fields in the first and second reference frame, and gives further definitions of various quantities. The coordinate systems (x, y, z) and (x', y', z') represent the first and second rotating frames, respectively.

We shall now mathematically treat with the motion of magnetization during the process described above. Since it is difficult to obtain directly the component of the magnetization in the x-y plane, we perform coordinate transformation twice. We start by putting the magnetization along H_e. During the time of which H_1 and H_a are applied simultaneously, the magnetization rotates as far as the angle $\alpha=\gamma H_e' t$ about H_e' by the effect of H_a in the second rotating frame. γ is the gyromagnetic ratio of proton nucleus. Then the components of the magnetization in this frame, (M_x', M_y', M_z'), can be written

$$\begin{pmatrix} M_x' \\ M_y' \\ M_z' \end{pmatrix} = M_0 \begin{pmatrix} \sin(\phi+\psi) & 0 & -\cos(\phi+\psi) \\ 0 & 1 & 0 \\ \cos(\phi+\psi) & 0 & \sin(\phi+\psi) \end{pmatrix} \begin{pmatrix} \sin(\phi+\psi) \\ \sin\alpha \cos(\phi+\psi) \\ \cos\alpha \cos(\phi+\psi) \end{pmatrix} . \tag{1}$$

After H_a is pulsed off, the magnetization further precesses about H_e as far as the angle $\beta=\gamma H_e t'$ by the effect of H_1 in the first rotating frame. At the end of this process the components of the magnetization become

$$\begin{pmatrix} M_x' \\ M_y' \\ M_z' \end{pmatrix} = M_0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\beta & -\sin\beta \\ 0 & +\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} \sin(\phi+\psi) & 0 & -\cos(\phi+\psi) \\ 0 & 1 & 0 \\ \cos(\phi+\psi) & 0 & \sin(\phi+\psi) \end{pmatrix} \times \begin{pmatrix} \sin(\phi+\psi) \\ -\sin\alpha \cos(\phi+\psi) \\ -\cos\alpha \cos(\phi+\psi) \end{pmatrix} . \tag{2}$$

Now, we shall transform (M_x', M_y', M_z') into the first rotating frame by using the following relation :

$$\begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix} = \begin{pmatrix} \cos\psi & 0 & -\sin\psi \\ 0 & 1 & 0 \\ \sin\psi & 0 & \cos\psi \end{pmatrix} \begin{pmatrix} M_x' \\ M_y' \\ M_z' \end{pmatrix} . \tag{3}$$

So that, the components of the magnetization in the first rotating frame are

$$\begin{pmatrix} M_x \\ \\ M_y \\ \\ M_z \end{pmatrix} = M_0 \begin{pmatrix} \begin{array}{l} \cos\psi[\sin^2(\phi+\psi)-\cos\alpha\cos^2(\phi+\psi)] \\ -\sin\psi[-\sin\alpha\sin\beta\cos(\phi+\psi)+(1-\cos\alpha)\cos\beta\sin(\phi+\psi)\cos(\phi+\psi)] \\ \quad -\sin\alpha\cos\beta\cos(\phi+\psi)-(1-\cos\alpha)\sin\beta\sin(\phi+\psi)\cos(\phi+\psi) \\ \sin\psi[\sin^2(\phi+\psi)-\cos\alpha\cos^2(\phi+\psi)] \\ +\cos\psi[-\sin\alpha\sin\beta\cos(\phi+\psi)+(1-\cos\alpha)\cos\beta\sin(\phi+\psi)\cos(\phi+\psi)] \end{array} \end{pmatrix} . \tag{4}$$

Suppose the resonance condition of rotary saturation, $\omega_a=\gamma H_e$, is satisfied, that is, $\phi+\psi=0$, Eq. (4) can be rewritten as follows :

$$\begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix} = M_0 \begin{pmatrix} -\sin\alpha\,\sin\theta + \sin\alpha\,\sin\beta\,\cos\theta \\ -\sin\alpha\,\cos\beta \\ -\cos\alpha\,\cos\theta - \sin\alpha\,\sin\beta\,\cos\theta \end{pmatrix}, \qquad (5)$$

where ϕ is replaced by θ by using the relation $\phi = \pi/2 - \theta$.

At this time we shall consider the effect of the relaxation. Relaxation takes place through the interactions of spins with each other and lattice. We denote the relaxation time due to these two interactions by T. Considering these relaxation effects, from Eq. (5) we can obtain the full representation for the observed magnetization in the laboratory frame.

$$M_{xy} = M_0\,[(\sin\alpha\,\sin\beta\,\cos\theta - \cos\alpha\,\sin\theta)^2 + (\sin\alpha\,\cos\beta)^2]^{1/2}\,\exp(-t/T), \qquad (6)$$

where M_{xy} is defined by $[M_x^2 + M_y^2]^{1/2}$. If we choose the time t' to be satisfied 90° pulse condition, that is to say $\beta = \pi/2$, Eq. (6) is rewritten

$$M_{xy} = M_0[\sin\alpha\,\cos\theta - \cos\alpha\,\sin\theta]\,\exp(-t/T). \qquad (7)$$

Noting that $\alpha = \gamma H_e t'$ and $H'_e = H_a \sin\theta$, we obtain the complete time development of the magnetization

$$M_{xy} = M_0\,[\cos\theta\,\sin(\gamma H_a t\,\sin\theta) - \sin\theta\,\cos(\gamma H_a t\,\sin\theta)]\exp(-t/T). \qquad (8)$$

Eq. (8) represents a damped oscillation of a variable t. In this equation, θ can be set up experimentally and M_0, T are measurable. Therefore by measuring M_{xy} for various time t and by comparison of the measured values of M_{xy} with Eq. (8), we decide the strength of the magnetic field produced by the audio coils.

III. Experimental results

Under the condition that $\gamma H_e t' = \pi/2$ and $\omega_a = \gamma H_e$, experiments have been performed throughout the observation of free induction signal following after H_1 pulse. Fig. 3 shows the results of the motion of the magnetization in the second rotating frame. In this figure, the normarized magnetization remaining after H_1 pulse is shown as the function of audio-pulse length. This figure represents the case of which the voltage applied across the audio coils to produce H_a is 3 volts and its frequency is 15.3 KHz. Solid line is drawn by Eq. (8) with $\theta = 99°$, $T = 5.0$ msec. T is fixed so as to fit each maximum points of theoretical values with experimental ones. There is no problems in doing so, since in determining H_a only the period of oscillation is needed. H_a is determined in gauss by adjusting so as to coincide the period of theoretical curve with the experimental points. In this case, we obtain that $H_a = 0.11$ G.

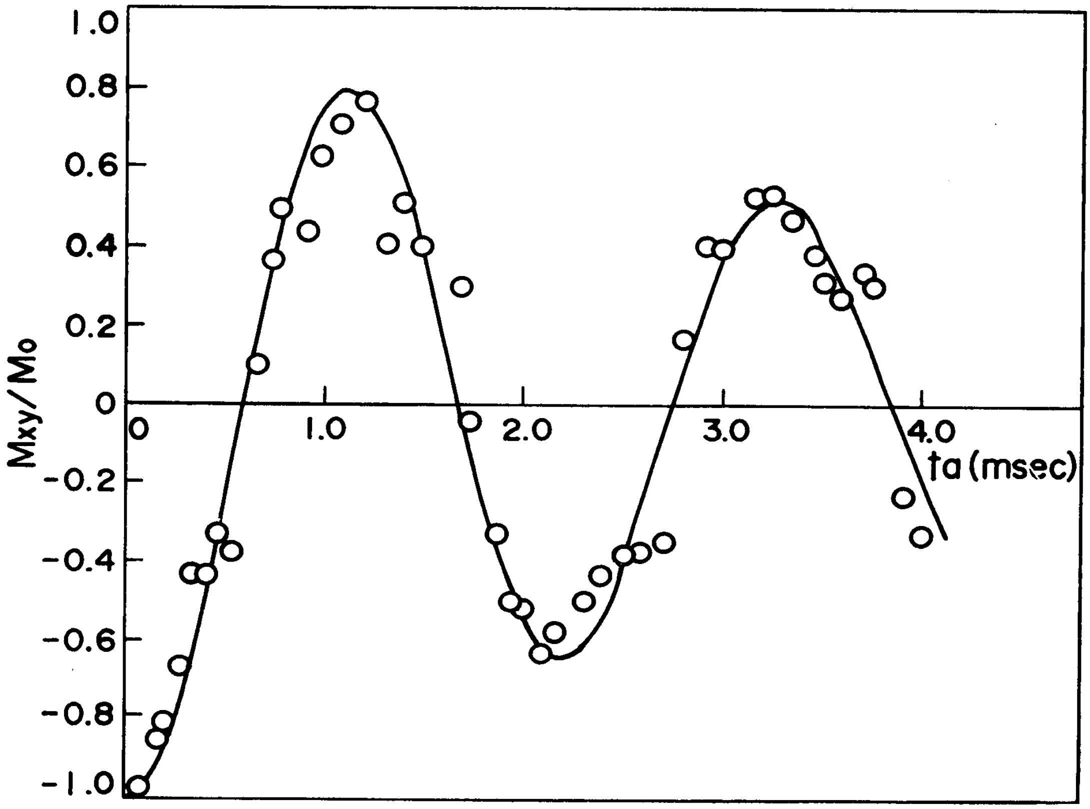

Fig. 3 The fraction of magnetizntion remaining after H_1 pulse as a function of audio-pulse length.

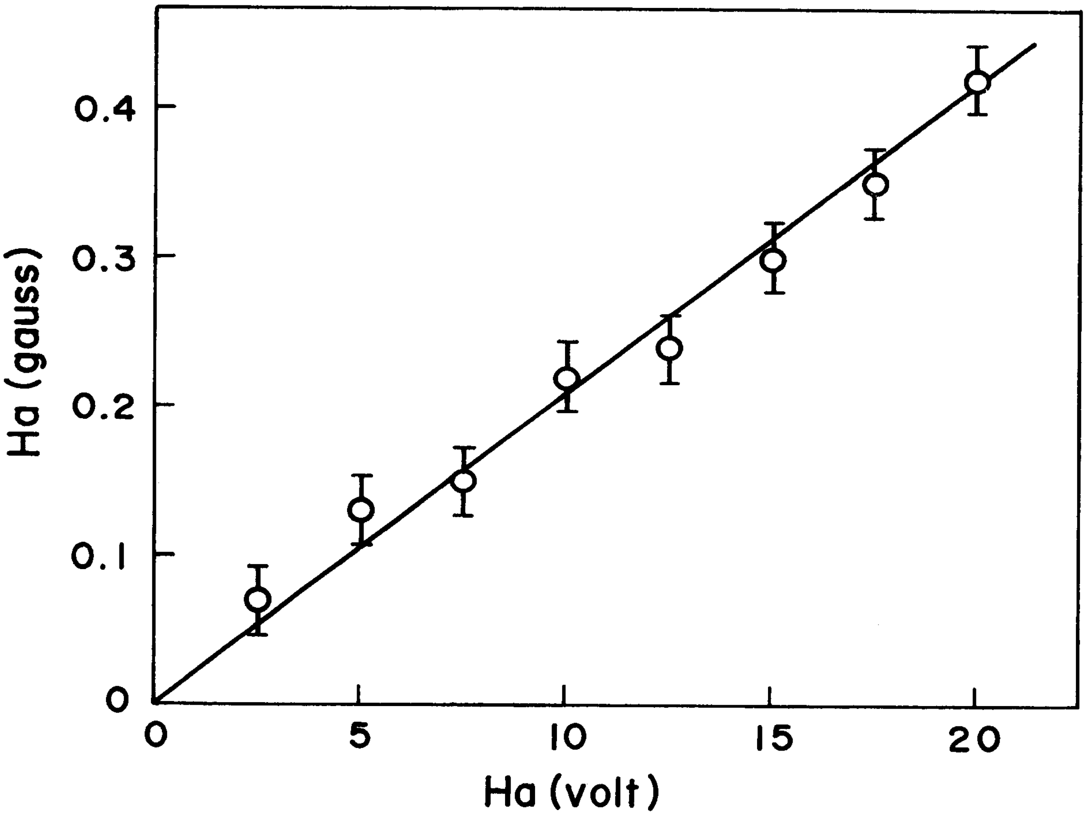

Fig. 4 The audio-field strength corresponding to the voltage across the audio coils.

In such a manner, we can obtain the values of magnetic field strength H_a corresponding to various voltages and various frequencies of the alternating current passes through the audio coils. By changing the voltage and the frequency of the alternating current independently each other and by repeating the above procedure, we can obtain the values of the magnetic field strength corresponding their voltages and frequencies. Fig. 4 shows the case of which the frequency of the audio field used is 16.6 KHz. Horizontal axis represents the voltage across the audio coils and longitudinal axis represents the strength of the magnetic field corresponding to this voltage. From Figs. 3 and 4, we may conclude that the error in determining the audio-field strength by our method is at most a few percents.

References

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3) J. R. Franz and C. P. Slichter: Phys. Rev. **148** (1968) 287.