Impedance-matching Section Using Exponential Line

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Abstract–When a tapered transmission line is used for the impedance-matching purpose, the matching characteristic depends remarkably upon the taper shape of the impedance level. The paper deals with the effect of the taper shape of a single or a cascaded exponential-tapered line section upon the impedance-matching characteristic.

I. INTRODUCTION

Electric power is transmitted most effeciently by a transmission line when no reflected wave is present. Thus, to obtain transmission of power with maximum effeciency, it is required to make the actual load impedance match to the characteristic impedance of the line.

At microwave frequencies, impedance matching is normally achieved by the aid of uniform transmission-line techniques, in which stub line and quarter-wave line techniques are included. An alternative type of impedance matching makes use of tapered line techniques. That is, a section of tapered transmission line acts as an impedance transformer at microwave frequencies. In the paper, we discuss the effect of the taper shape of impedance level upon the impedance-matching characteristic when a single or a cascaded exponential-tapered line is used for the impedance-matching purpose.

II. MATCHING CHARACTERISTICS

In the chain parameters of any tapered lossless transmission line of finite length, both of $A(j\omega)$ and $D(j\omega)$ are real values, while $B(j\omega)$ and $C(j\omega)$ are imaginary values. When a length l of a tapered line in which the impedance level $W(x)$ depends on the length x measured from the generator end of the line is used for the purpose of matching the actual load impedance R_L to the generator impedance R_G, the standing-wave ratio on the generator side of the tapered line can be expressed by [1]

$$\text{Standing-wave ratio} = (1+|\rho|)/(1-|\rho|) \tag{1}$$

where

$$|\rho| = \sqrt{\frac{\{(a-d)\cosh\tau+(a+d)\sinh\tau\}^2+\{(b-c)\cosh\kappa+(b+c)\sinh\kappa\}^2}{\{(a+d)\cosh\tau+(a-d)\sinh\tau\}^2+\{(b+c)\cosh\kappa+(b-c)\sinh\kappa\}^2}}$$

$$\tau = \tfrac{1}{2}\ln\{W(0)R_L/W(l)R_G\}$$

$$\kappa = \tfrac{1}{2}\ln\{W(0)W(l)/R_GR_L\}$$

$$a = A\sqrt{W(l)/W(0)}$$

$$b = -jB/\sqrt{W(0)W(l)}$$

$$c = -jC\sqrt{W(0)W(l)}$$

$$d = D\sqrt{W(0)/W(l)}$$

The τ and κ denote the ratio factor and the level factor, respectively. The a, jb, jc, and d are the normalized chain parameters, that is, they are the chain parameters of a two-port network which is composed of an ideal transformer of turns ratio $1/\sqrt{W(0)}$, the tapered section, and another ideal transformer of turns ratio $\sqrt{W(l)}$ which are connected in cascade so that each of the impedance levels at both ports of the network is normalized to 1 ohm.

All of the ratio factor, the level factor, and the normalized chain parameters will change when the taper shape of the impedance level of a tapered section is changed. Thus, the taper shape has a direct effect upon the impedance-matching characteristic, for it depends the normalized chain parameters and so on.

The value of the level factor κ can be chosen independently to the normalized chain parameters. If the impedance level of the tapered matching section increases uniformly by a factor α, then the value of κ increases by a constant $\ln\alpha$, but the ratio factor and the normalized chain parameters are kept unchanged. Thus, the tapered sections having different values of κ will have familiar types of impedance-matching characteristic. An example of the case using a single exponential-tapered line having different value of κ is shown in Fig. 1. In the case, the frequencies at which the standing-wave ratio is unity are independent of κ. The standing-wave ratio increases as only the absolute value of κ increases. Thus, the effective pass-band width becomes narrower as the absolute value of κ increases. It follows from these that $\kappa=0$ for the best impedance matching by κ.

The value of the ratio factor τ has direct effects upon the normalized chain parameters. Thus, the frequencies at which the standing-wave ratio is minimum

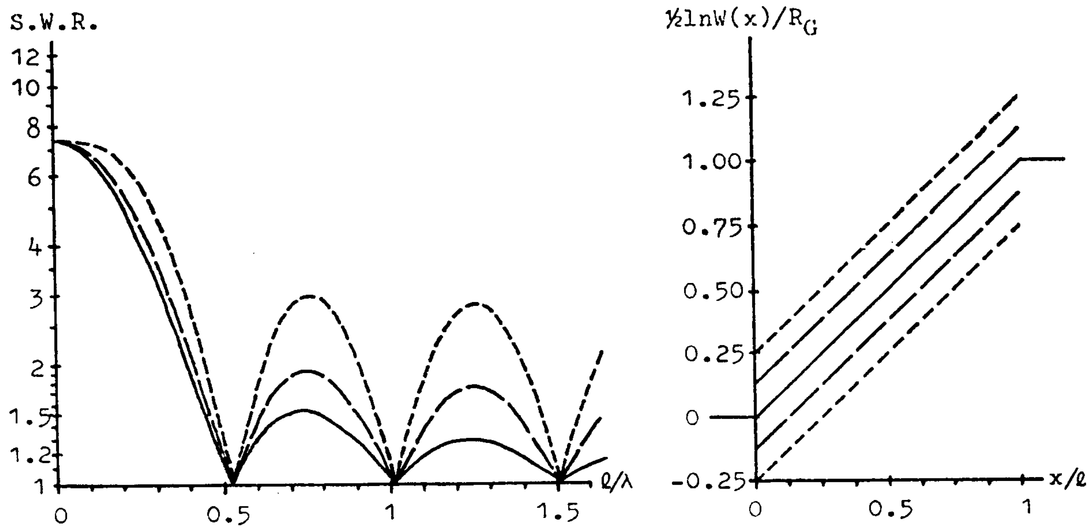

Fig. 1. Relationship between standing-wave ratio curves and taper shapes in the case of $R_L/R_G=e^2$ and $\tau=0$.

will shift remarkably when the value of τ is changed. We consider the case that the value of τ is changed under the condition in which $\kappa=0$. An example of the case using a single exponential-tapered line having different value of τ is shown in Fig. 2. In the case, the effective pass-band width becomes narrower as the absolute value of τ increases. It follows from this that $\tau=0$ for the best impedance matching by τ. However, a non-zero value of τ should be chosen when a short line length is especially required. That is, a large positive value of τ should be chosen when $R_G<R_L$, while a large negative value of τ does when $R_G>R_L$.

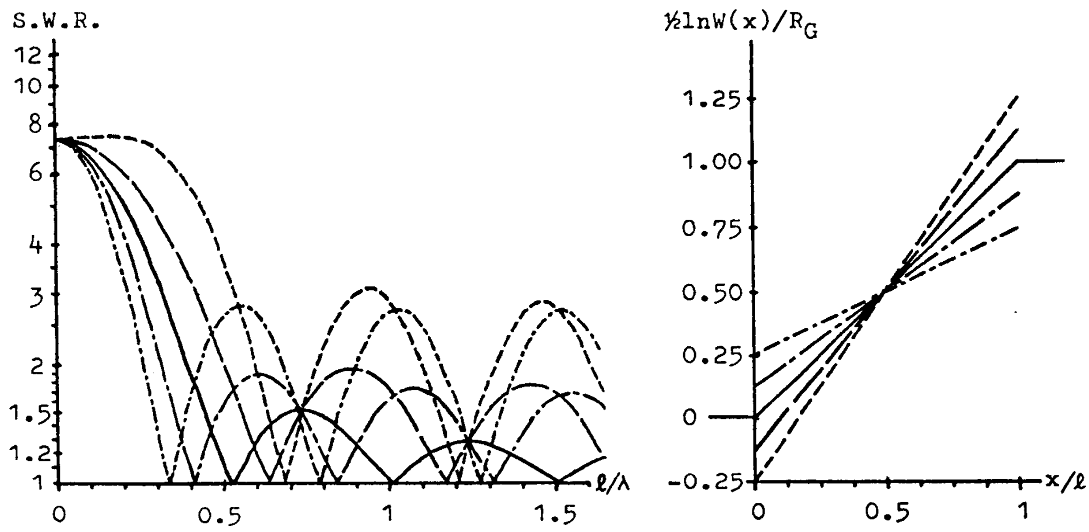

Fig. 2. Relationship between standing-wave ratio curves and taper shapes in the case of $R_L/R_G=e^2$ and $\kappa=0$.

From these considerations it follows that a pair of conditions,

$$\kappa = 0 \quad \text{and} \quad \tau = 0,$$

or

$$W(0) = R_G \quad \text{and} \quad W(l) = R_L, \qquad (2)$$

should be satisfied for the best impedance-matching purpose unless a short line length is especially required. Under the conditions in (2) the standing-wave ratio is unity at only the frequencies at which $a=d$ and $b=c$. An example of the case using two exponential-tapered lines in cascade under the conditions in (2) is shown in Fig. 3.

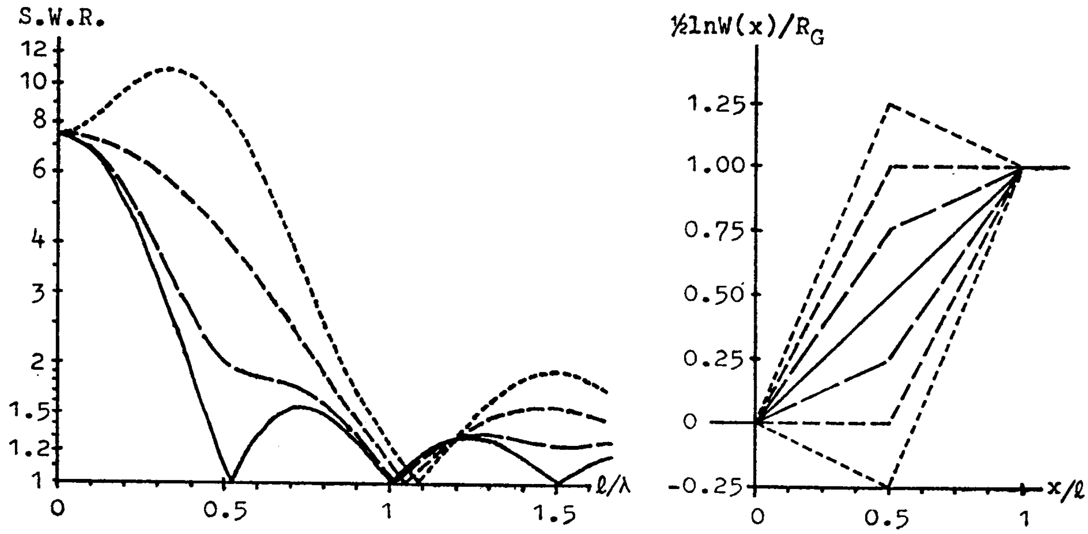

Fig. 3. Relationship between standing-wave ratio curves and taper shapes in the case of $R_L/R_G = e^2$, $\kappa=0$, and $\tau=0$.

As being pointed out in the previous paper [1], the impedance-matching characteristic is kept unchanged in the case of $\kappa=0$ when a tapered matching section is replaced by the new section adjoint to the original.

III. Conclusion

In all cases an impedace-matching section should be chosen of such a condition that $\kappa=0$ for the best impedance-matching purpose. Moreover, the condition $\tau=0$ should be added to the condition $\kappa=0$, that is, a pair of the conditions in (2) should be satisfied, unless a short line length is especially required. The impedance matching section using a single exponential-tapered line with the conditions in (2) has the best matching characteristic when the taper shape is restricted to be exponential. However, the discussion about the taper shapes other than the exponential taper is the subject for a future study.

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REFERENCE

[1] Y. Nakajima, "Adjoint transmission-line sections," Bulletin, Okayama College of Science, No. 10, pp. 139-143, 1974