

PARTICLE RATIOS IN PROTON-PROTON COLLISIONS AND SCALING PART OF THE WHOLE REGION FORMULA

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Abstract

Inclusive reaction $p + p \rightarrow c + X$ is analyzed with the whole region formula. We determine the power of scaling part from the analysis of particle ratios phenomenologically. The different dynamical parameters γ for mesons and baryons are obtained and the newly discovered heavy particle production rates are also discussed.

§ 1. Introduction

The single-particle inclusive reactions have been extensively investigated by a number of experimental groups. The data of two-particle distributions are also widely accumulated and provide us important informations about the dynamics underlying in hadron reactions. The whole region formula¹⁾ for one-particle distribution function and its generalization to n-particle distribution function as the bond model²⁾ were proposed by K. Kinoshita and H. Noda. We are now much interested in applying the model to the two-particle spectra.

In the previous papers³⁾ we have analyzed the inclusive reaction $p + p \rightarrow \pi + X$ with the whole region formula which exhibits Mueller Regge behaviour at low transverse momentum (p_T) and power behaviour at high p_T . A good agreement with the pion data was obtained. The dynamical parameter γ ³⁾ incorporates the soft collision at low p_T regions and hard collision at high p_T ones. Recent field theoretical approaches for urbaryons⁴⁾ lead to the power behaviour at large transverse momentum, supporting this model. The purpose of this note is to analyze one particle distribution spectra, the particle ratios and the fractions with the whole region formula and to determine the power of the scaling part. We present the newly discovered J(3.1), $\psi(3.7)$ ⁵⁾ and D(2.0)⁶⁾ production rates. This is the preliminary work for clarifying the hadrodynamics by the bond model, using two-particle data.

In the next section we summarize kinematics and the formula. Comparison with experiments will be given in § 3 and discussions in § 4.

§ 2. Kinematics and formula

We consider the reaction $a+b \rightarrow c+X$. Let p_a , p_b , and $p_c (\equiv (E, p_T, p_{\parallel}))$ denote the four-momenta of particles a , b and c in the center-of-mass system respectively. The Lorentz invariant variables are defined as follows;

$$\begin{aligned} s &= (p_a + p_b)^2, \quad t = (p_a - p_c)^2, \quad u = (p_b - p_c)^2, \\ M_X^2 &= (p_a + p_b - p_c)^2 = s + t + u - m_a^2 - m_b^2 - m_c^2, \\ t^* &= 2p_a \cdot p_c = m_b^2 + m_c^2 - t, \quad u^* = 2p_b \cdot p_c = m_b^2 + m_c^2 - u, \\ x_1 &= t^*/s, \quad x_2 = u^*/s. \end{aligned} \quad (1)$$

Feynman scaling variable and energy fraction are given as $x = 2p_{\parallel}/\sqrt{s}$ and $\bar{x} = 2E/\sqrt{s}$. For the analysis of high energy collisions it is convenient to use light-like fractions $x_{\pm} = (\bar{x} \pm x)/2$. In high energy limit we have the following approximate relations,

$$\begin{aligned} t^* &\sim (E - p_{\parallel})\sqrt{s}, \quad u^* \sim (E + p_{\parallel})\sqrt{s}, \\ \frac{t^*u^*}{s} &\sim m_T^2 = m_c^2 + p_T^2, \\ M_X^2 &\sim s - 2E\sqrt{s}, \\ x_+ &\sim x_1, \quad x_- \sim x_2. \end{aligned} \quad (2)$$

With the light-like fractions, the whole region formula for $a+b \rightarrow c+X$ is written as follows¹⁾;

$$\begin{aligned} \rho_{ab^c} &\equiv E \frac{d\sigma}{d^3p_c} = g_0 \sum_{\nu} n_{ab^c}(\nu) g_{ab^c}(\nu) \bar{\rho}_{ab^c}(\nu), \\ \bar{\rho}_{ab^c}(\nu) &= \frac{1}{1-x_+-x_-} [F(m_T^2) f(x_+, x_-)]^{2r(p_T)}, \\ F(m_T^2) &= [(m_T^2 + c/\sqrt{s})/m_0]^{-L}, \\ f(x_+, x_-) &= x_+^{n_{ab}+n_{ac}} \left(\frac{1-x_+-x_-}{1-x_-} \right)^{n_{aX}} (1-x_+-x_-)^{n_{cX}} \\ &\quad \cdot \left(\frac{1-x_+-x_-}{1-x_+} \right)^{n_{bX}} x_-^{n_{ab}+n_{bc}}, \end{aligned} \quad (3)$$

where ν denotes a type of production mechanism of particle c . The summation runs all over the types which contribute to the process. The type ν is specified by a set of $\{n_{ij}\}$, where n_{ij} is the number of valence urbaryon lines connecting i and j particles ($i, j = a, b, c$ and X). The numbers of valences in a meson and a baryon are two and three, respectively and that of X state is defined by $n_X = n_{aX} + n_{bX} + n_{cX}$. Here we have taken the viewpoint that hadrons are made of valence urbaryons and

Tab. 1

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sea ones. In high energy interactions the role of compositeness becomes more important, however, as a whole this effect is concentrated on a few valence urbaryon flows. The number $n_{ab}(\nu)$ is the contributing factor of ν type to the process and determined from the simple counting rule²⁾. Their values for $p+p \rightarrow c+X$ in quartet quark model are listed in Tab. 1. $g_{ab}(\nu)$ is the coupling constant of ν type production mechanism and g_0 is the overall normalization constant.

For example, $\rho_{pp}^{K^+}$ consists of three components,

$$\rho_{pp}^{K^+} = g_0 g_{pp}^K \{ 2\bar{\rho}_{pp}^{K^+}(H \otimes P_a) + \bar{\rho}_{pp}^{K^+}(H \otimes P_b) + \bar{\rho}_{pp}^{K^+}(P \otimes P) \},$$

where a set $\{n_{ij}\}$ is expressed by a bond diagram which is illustrated in Fig. 1. From $p+p \rightarrow \pi+X$ we obtained the ν independence of $g_{pp}(\nu)$, so this is assumed for other particles in proton-proton collisions. It is noted that the power of scaling part $f(x_+, x_-)$ is well determined from the particle ratio data.

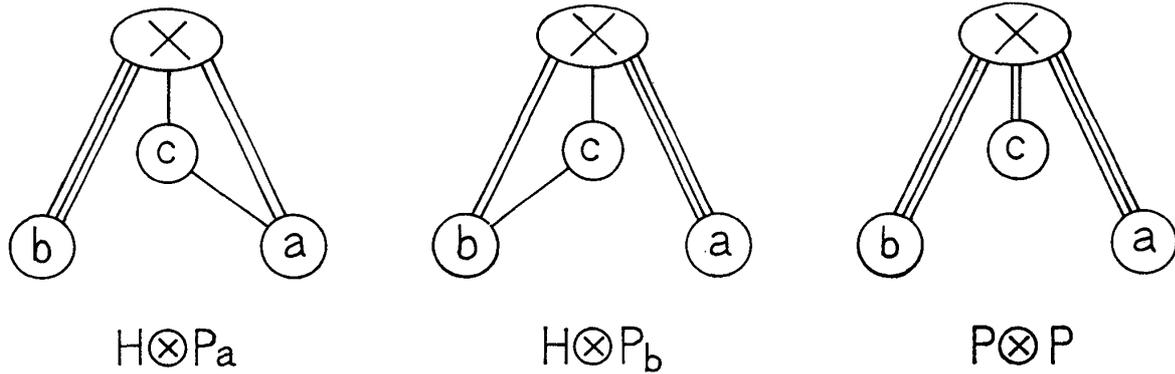


Fig. 1; The urbaryon bond diagram for $p+p \rightarrow meson+X$.

When the longitudinal momentum of particle c is equal to zero, the ratio of $H \otimes P$ type to $P \otimes P$ one for $p+p \rightarrow meson+X$ is given as follows;

$$\left[\frac{\bar{\rho}_{pp}^{meson}(H \otimes P_a) + \bar{\rho}_{pp}^{meson}(H \otimes P_b)}{\bar{\rho}_{pp}^{meson}(P \otimes P)} \right]_{p_T=0} = \frac{4m_T(\sqrt{s}-2m_T)}{\sqrt{s}-4m_T} \quad (4)$$

This factor is the origine of particle/anti-particle production ratios at $p_T=0$. It is an increasing function of m_T and decreasing one of \sqrt{s} , explaining the ratios of π^+/π^- and K^+/K^- qualitatively. Quantitatively, however, Eq. (3) yields rather small K^+/K^- ratio at $p_T \leq 3\text{GeV}/c$ in comparison with the FNAL data. On the other hand, the value of $\gamma(p_T)$ at small p_T region ($p_T \leq 1\text{GeV}/c$) is nearly equal to 0.5 from previous analysis.³⁾ We adopt the following expression of $\rho_{ab}(\nu)$ in terms of products of Lorentz invariants,²⁾

$$\begin{aligned} \bar{\rho}_{pp}(\nu) &= \{F(m_T^2)\}^{-2\tau(p_T) \cdot N/4} M_X^2 \frac{s}{M_X^2} f, \\ F(m_T^2) &= \left\{ \left(\frac{t^* u^*}{s} + \frac{0.25}{\sqrt{s}} \right) / 1.4 \right\}, \end{aligned} \quad (5)$$

$$\begin{aligned}
 f = & \left(\frac{m_c^2 + M_X^2}{s} \right)^{n_{cX}} \left(\frac{m_a^2 + m_b^2}{s} \right)^{n_{ab}} \left(\frac{t^* u^* / s}{m_a^2 + m_b^2} \right)^{n_{ab}} \left(\frac{m_b^2 + M_X^2}{m_b^2 + M_X^2 - t} \right)^{n_{bX}} \\
 & \cdot \left(\frac{t^* u^* / s}{m_a^2 + m_c^2} \right)^{n_{ac}} \left(\frac{m_a^2 + m_c^2}{m_a^2 + m_c^2 - t} \right)^{n_{ac}} \left(\frac{m_a^2 + M_X^2}{m_a^2 + M_X^2 - u} \right)^{n_{aX}} \left(\frac{t^* u^* / s}{m_b^2 + m_c^2} \right)^{n_{bc}} \\
 & \cdot \left(\frac{m_b^2 + m_c^2}{m_b^2 + m_c^2 - u} \right)^{n_{bc}},
 \end{aligned}$$

where the approximation (2) is assumed. The main difference between Eqs. (3) and (5) is the power of the scaling part $f(x_+, x_-)$ i.e. in Eq. (5) 2γ of the corresponding part is equal to one.

§ 3. Comparison with experiments

In this section we examine the quantitative aspects of Eq. (5). First we consider the p_T dependence of particle/anti-particle ratios. From Eq. (5) we have the ratio of $H \otimes P$ to $P \otimes P$ type for mesons at $p_{\neq} = 0$ as

$$\begin{aligned}
 [R^{meson}]_{p_{\neq}=0} &= \left[\frac{\bar{\rho}_{pp}^{meson}(H \otimes P_a) + \bar{\rho}_{p\bar{p}}^{meson}(H \otimes P_b)}{\bar{\rho}_{pp}^{meson}(P \otimes P)} \right]_{p_{\neq}=0} \\
 &= \frac{s}{s - 2\sqrt{s} m_T + 2m_c^2} \frac{s}{s - \sqrt{s} m_T} \frac{2m_T}{\sqrt{s}}.
 \end{aligned}$$

The ratio R^{meson} and the corresponding factor for baryons R^{baryon} determine the ratio of particle/anti-particle;

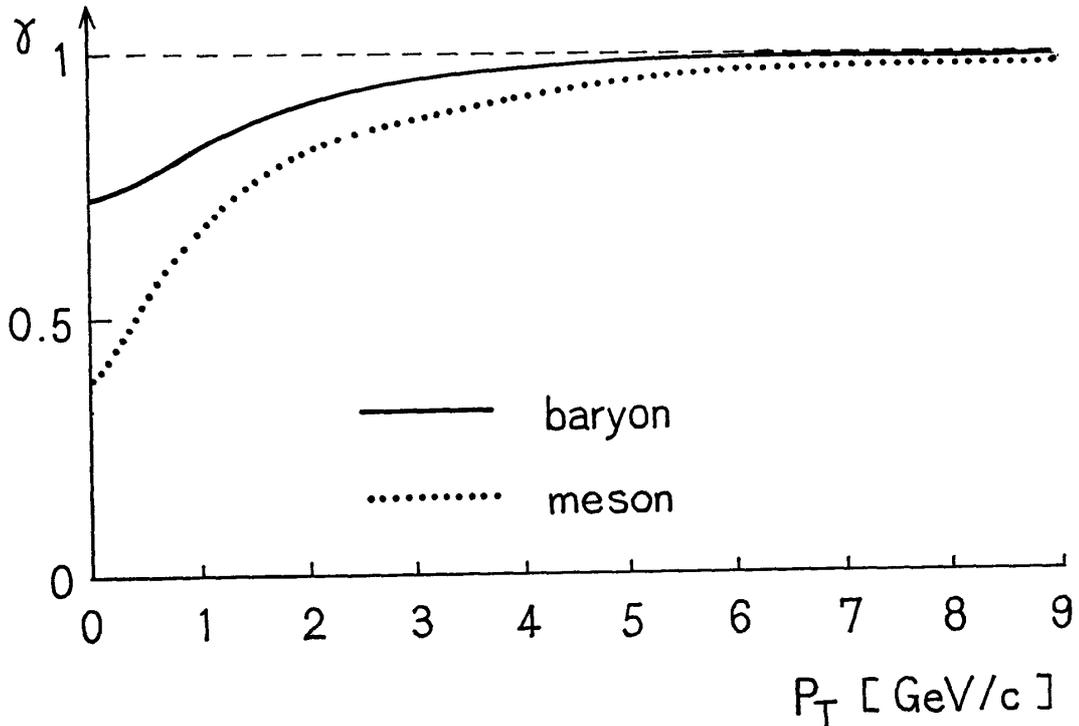


Fig. 2; The p_T dependence of the dynamical parameter $\gamma(p_T)$. The dotted line is for mesons and the solid one for baryons.

$$\pi^+/\pi^- = 1 + \frac{R^\pi}{1+R^\pi}, \quad K^+/K^- = 1 + 2R^K.$$

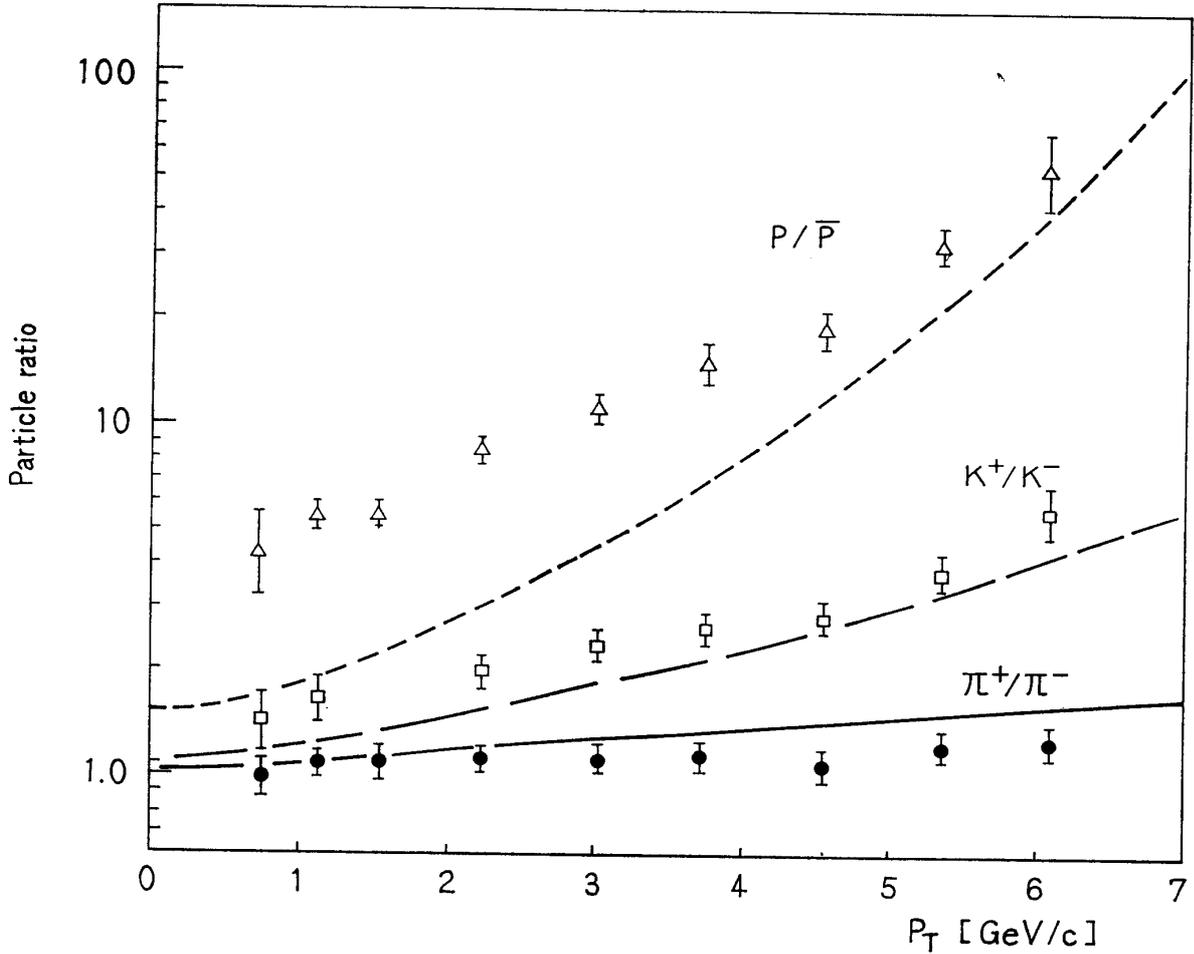
The results are shown in Fig. 3_a. To get other particle ratios and absolute values of spectra we have to set the values of the parameters. The values of g_{pp^c} are listed in Tab. 2, where r_λ (r_c) is the coupling constant of λ quark (charm quark) to the Pomeron. When p_T is large, mass effect can be neglected and $r_\lambda \sim K^+/\pi^+$ and $r_c/r_\lambda \sim D^0/K^-$. The parameters $\gamma(p_T)$ for mesons and baryons are shown in Fig. 2. Taking the following values

$$g_0 = 0.3 \text{ mb GeV}^{-2} c^3, \quad r_\lambda = \frac{1}{2}, \quad r_c = \frac{1}{20}$$

$$N=8, \quad \gamma(p_T) = 1 - \frac{\exp(-0.21 m^2 p_T)}{1.43 + m^2 p_T} \quad \text{for mesons,}$$

$$N=9, \quad \gamma(p_T) = 1 - \frac{\exp(-0.47 m^2 p_T)}{1.40 + m^2 p_T} \quad \text{for baryons,}$$

we get other particle ratios as shown in Figs. 3_b and 3_c. Comparison with particle fraction data at 300 and 1500 GeV/c⁰ are given in Figs. 4_a and 4_b.

Fig. 3_a

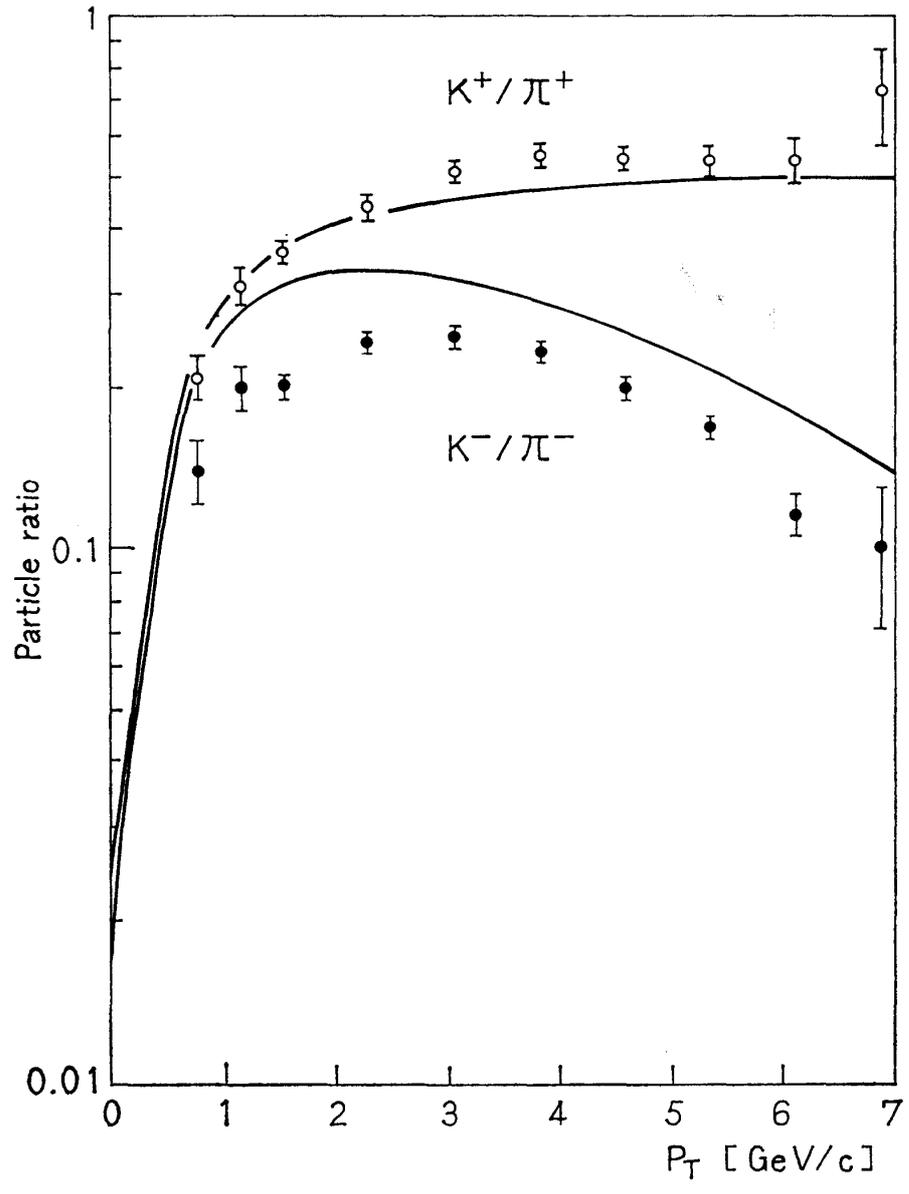


Fig. 3b

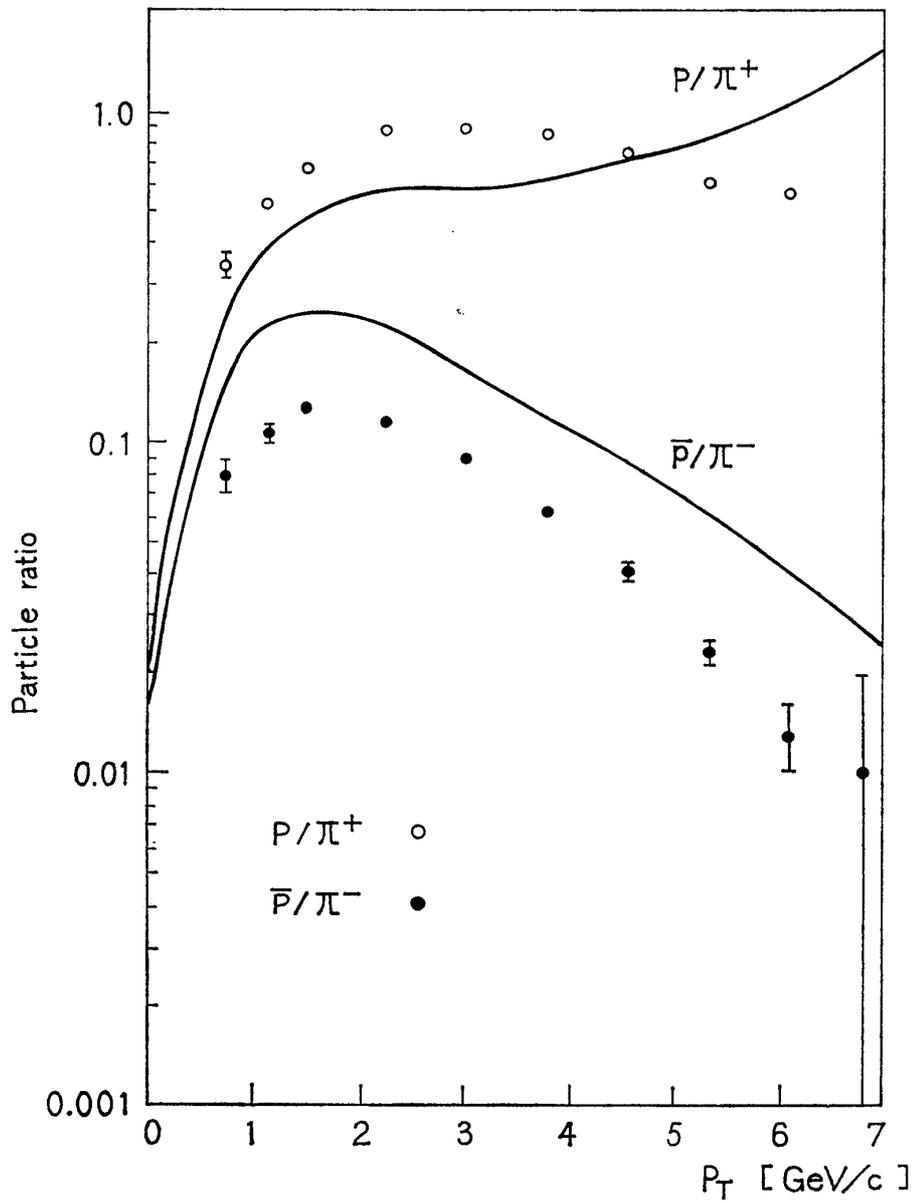


Fig. 3c

Fig. 3; The particle ratios at $p_{\parallel}=0$ of a) p/\bar{p} , K^+/K^- and π^+/π^- b) K^+/π^+ and K^-/π^- c) p/π^+ and \bar{p}/π^- for $p_L=300$ GeV/c. The data are taken from the ref. 8).

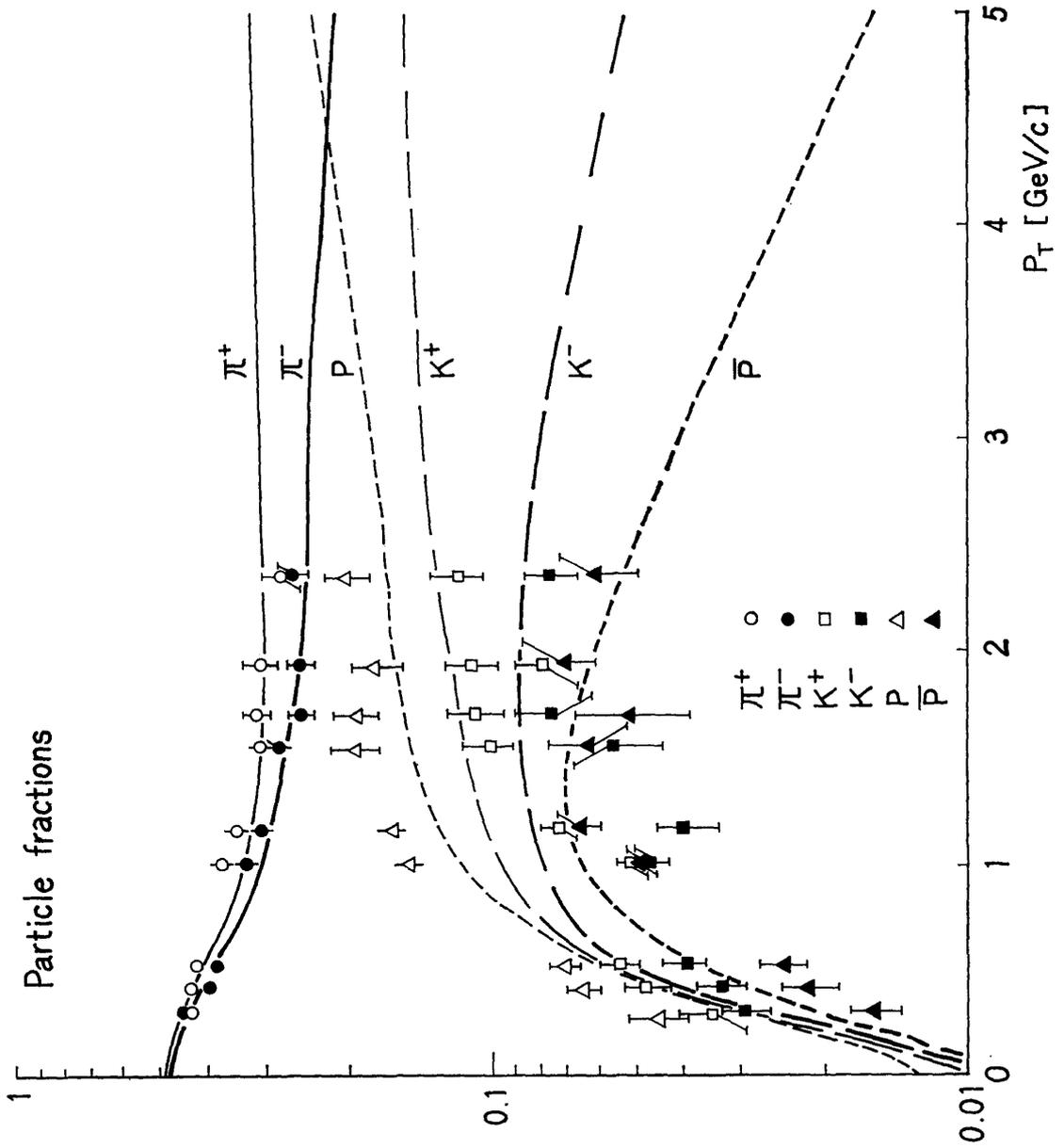


Fig. 4a

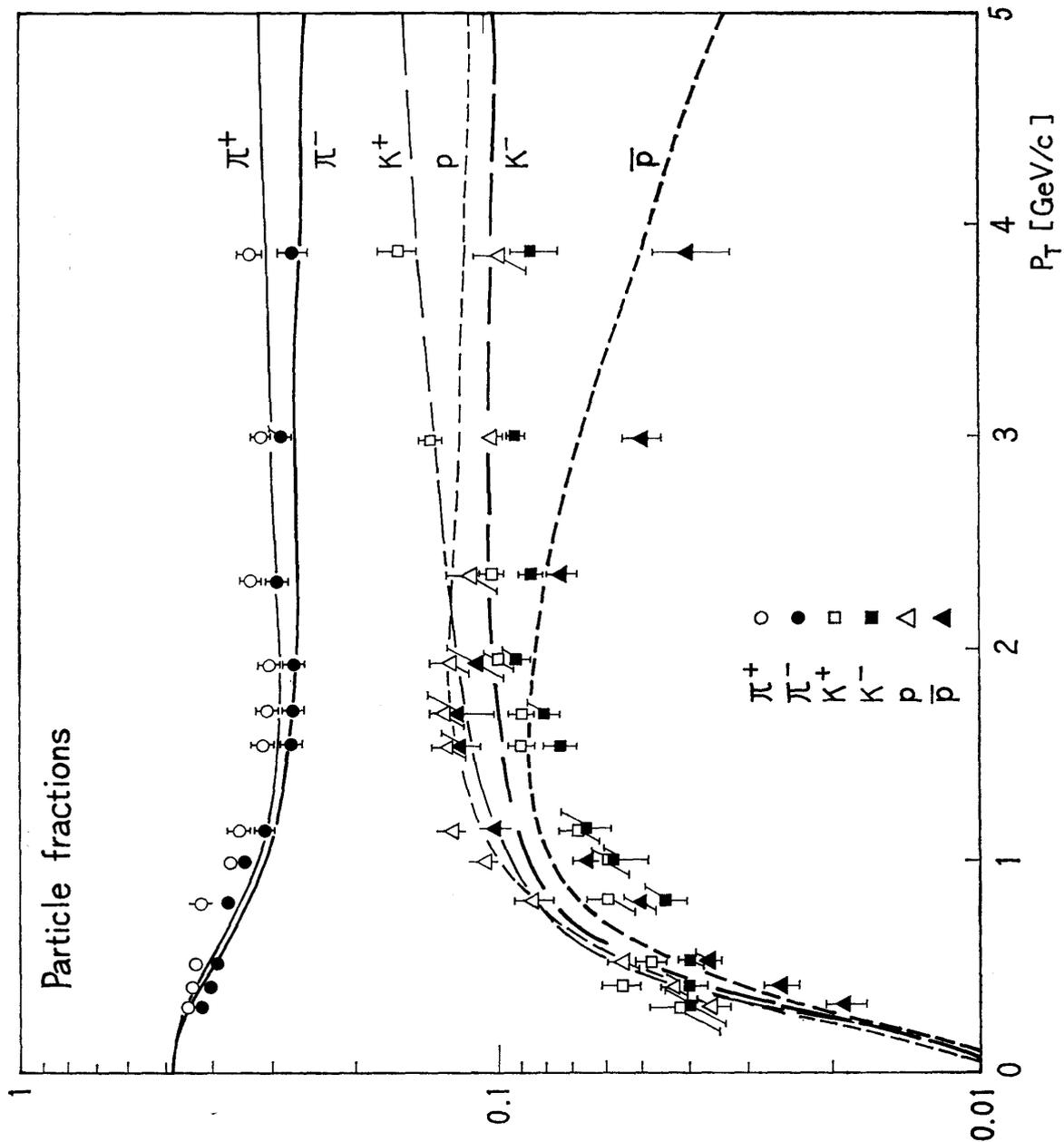
Fig. 4_b

Fig. 4; The p_T dependence of the charged particle composition at $p_{\perp} = 0$ for a) $p_L = 300$ GeV/c and b) $p_L = 1500$ GeV/c. The data are taken from ref. 9).

From Figs. 3 and 4, it seems to us that the scaling part of Eq. (5) is a fairly good one. Transverse momentum dependence of ρ_{pp}^c at $p_{\perp} = 0$ is given in Fig. 5_a and rapidity dependence at $p_T = 0.4$ GeV/c is in Fig. 6_a for $p_L = 1500$ GeV/c. Fig. 6_a shows discrepancies of proton spectrum at fragmentation region. Contribution from $H \otimes P$ type is estimated so small and the problem of ν dependence of $g_{pp}^c(\nu)$ is an open question.

Finally in Figs. 5_b and 6_b, we present the production rates of newly discovered

particles. Due to the small coupling constant r_c and small mass of Niu particle, copious production rates of D^0 and \bar{D}^0 in comparison with J(3.1) are predicted. Discussions about the estimation of r_s and r_c are given in reference 11).

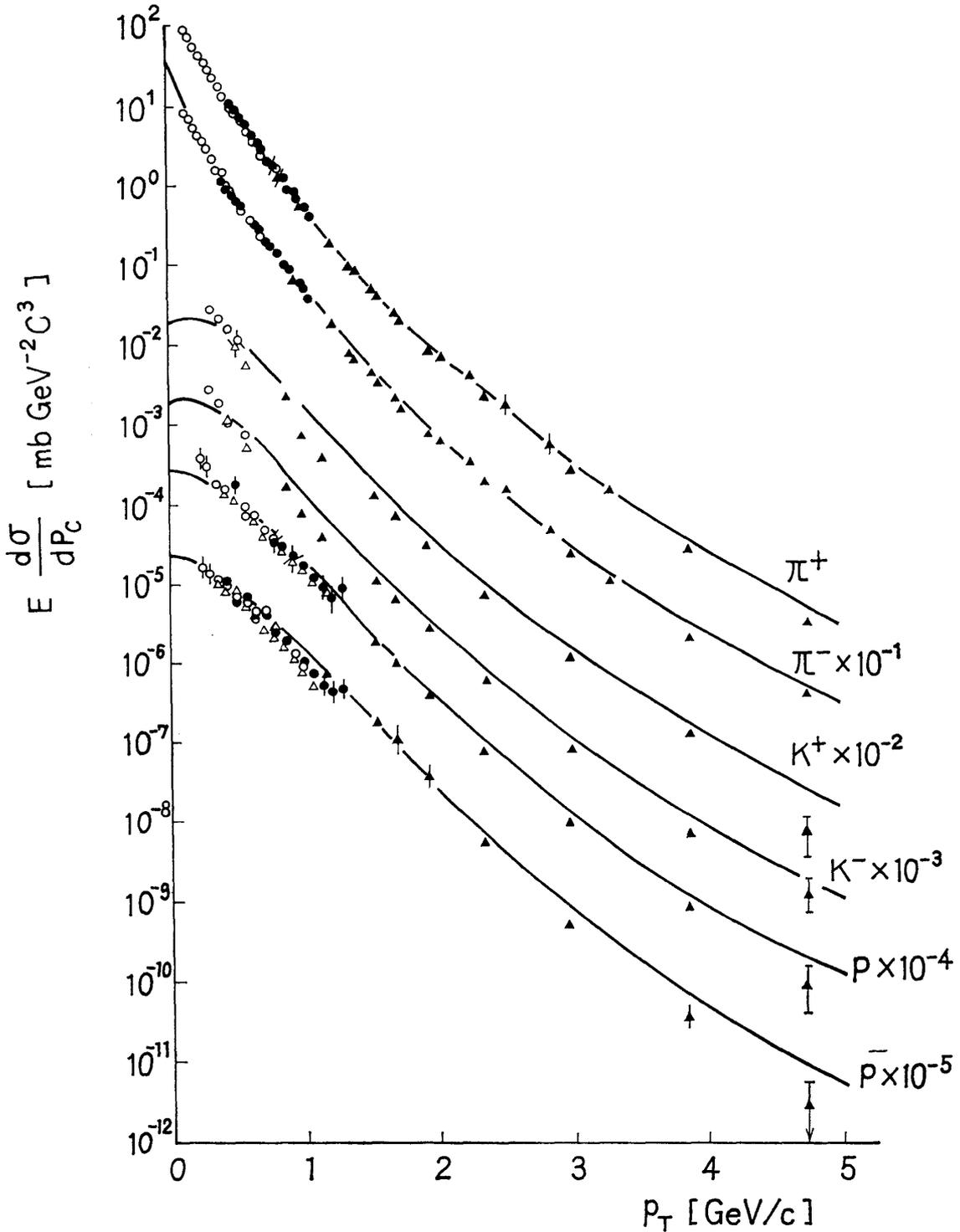


Fig. 5a; The p_T dependence of one-particle distribution function of pions, kaons, protons and anti-protons at $p_L=1500 \text{ GeV}/c$ and $p_{\perp}=0$. The data are taken from the ref. 9).

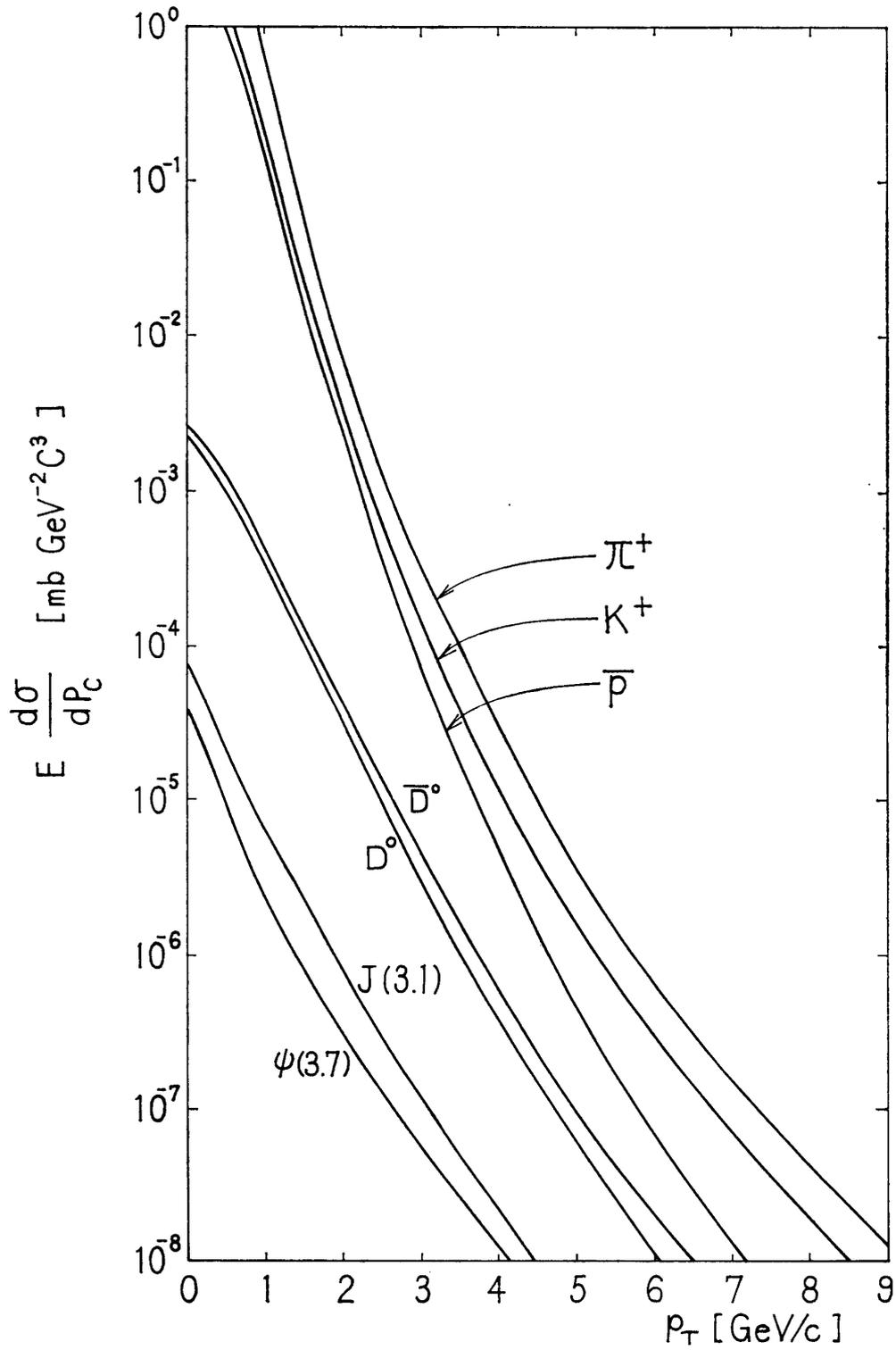


Fig. 5_b; As in Fig. 5_a including the new heavy particles $\bar{D}^0(2.0)$, D^0 , $J(3.1)$ and $\psi(3.7)$.

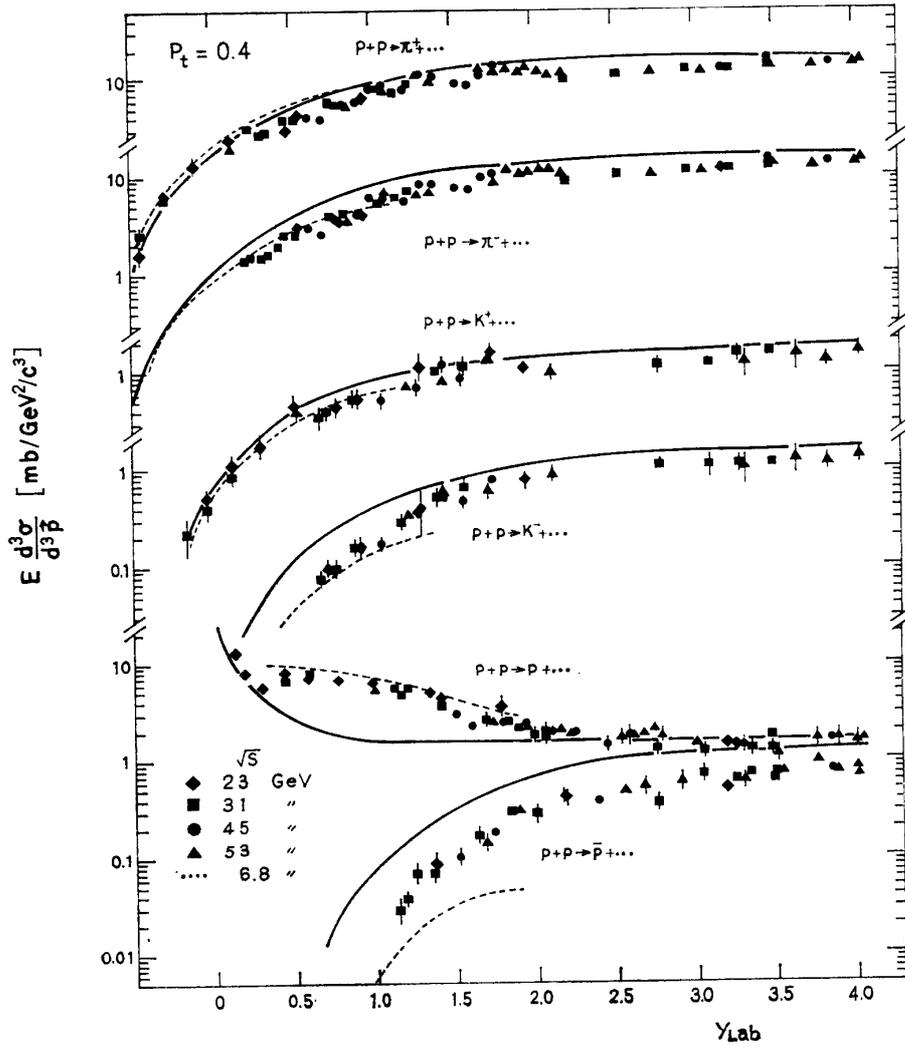


Fig. 6a; The y_{Lab} dependence of the one-particle distribution function of pions, kaons, protons and anti-protons at $p_L=1500$ GeV/c and $p_T=0.4$ GeV/c. The data are taken from the ref. 10).

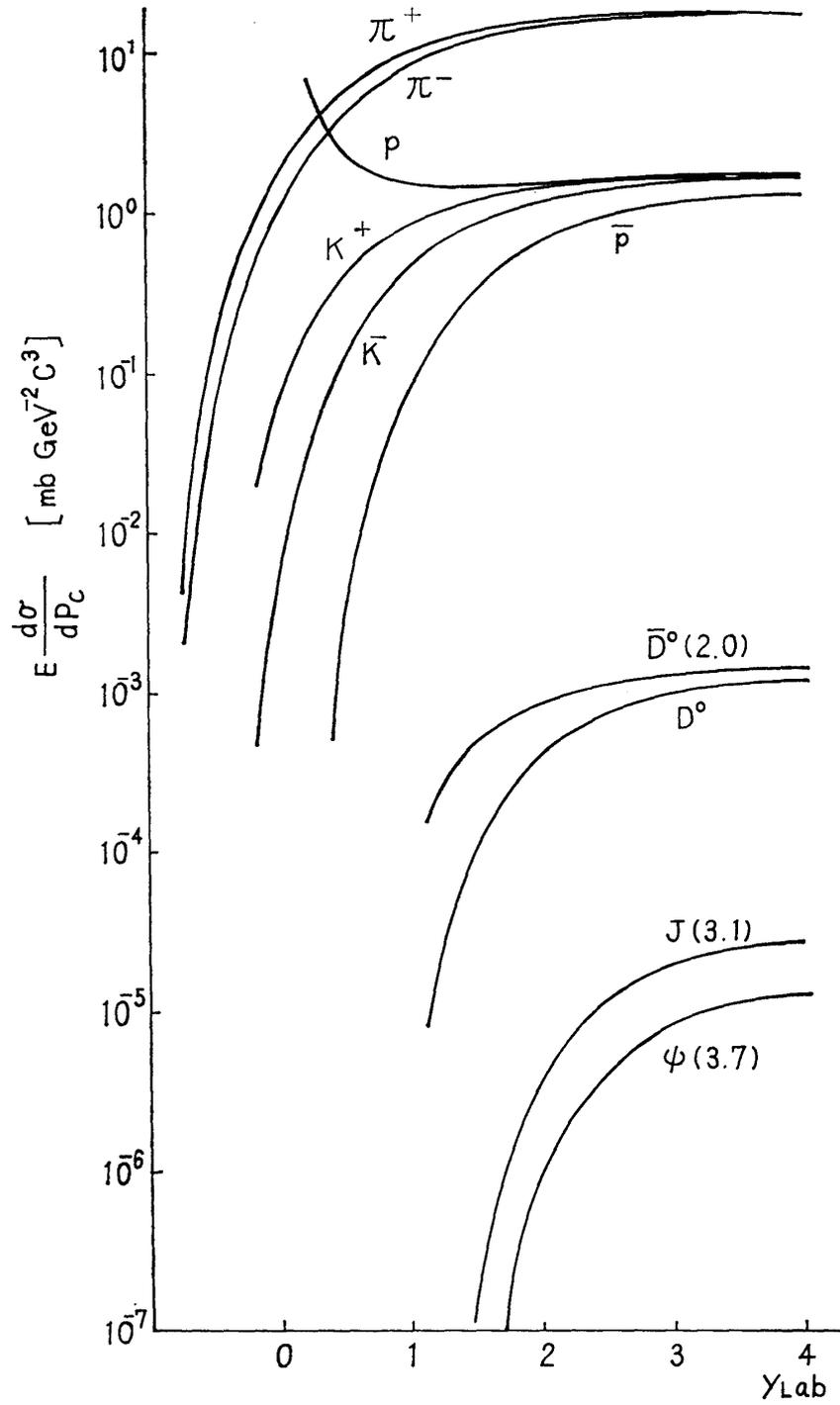


Fig. 6_b: As in Fig. 6_a including the new heavy particles $\bar{D}^0(2.0)$, D^0 , $J(3.1)$ and $\phi(3.7)$.

§ 4. Discussions

As seen above, a fairly good agreement of our model with experiments was obtained. Especially from the particle ratios, the power of scaling part of the whole region formula was well determined. In one-particle distribution phenomena there are several different aspects between low and high p_T regions. In Eq. (5) these differences are attributed to the dynamical parameter γ . In this note we used the invariant form for the scaling part both at small and large p_T regions. Discrepancies of p/\bar{p} and K^+/K^- ratios at small p_T suggest the different scaling form for K^+ , K^- , p and \bar{p} at $p_T \leq 1 \text{ GeV}/c$ from the model used here. It is possible to parametrize

$$\rho_{pp^c}(\nu) = F(m_T^2)^{-N_T} (\nu_T)^{1/2} \frac{s}{M_T^2} f^{\gamma'},$$

$\bar{\rho}_{pp^c}(\nu)$ as where $\gamma' \sim 0.5$ at large p_T regions and $2\gamma' < 1$ at small p_T regions. This will explain larger K^+/K^- and p/\bar{p} ratios at $p_T \leq 1 \text{ GeV}/c$. For pions f in Eq. (5) seems to be a good function. Rapidity dependence at $p_T = 0.4 \text{ GeV}/c$ is grossly available and asserts the simple counting rule, but in detail ν dependence of $g_{ab^c}(\nu)$ must be taken into account of as noted in previous section. Discrepancy of p/π^+ ratio in Fig. 3. at large p_T may be recovered by taking the value of N for baryons equal to 12 as FNAL data. More careful analyses including the particle ratios at various scattering angles are required in order to check the form of f .

We may give a following understanding about the parameter γ . As shown in Fig. 2, γ of baryons is larger than that of mesons. This would be due to the different interaction of valence urbaryons with other sea ones between for mesons and baryons. At large p_T where hard collisions occur, this effect disappears and $\gamma \approx 1$.

Since $D^0 \sim c\bar{p}_0$ and $D^+ \sim c\bar{n}_0$, both $H \otimes P$ and $P \otimes P$ types contribute to \bar{D}^0 and D^- and only $P \otimes P$ type does to D^0 and D^+ . So we have $D^- \geq D^+$ in contrast to $K^+ \geq K^-$. Due to the large mass of Niu particle, $H \otimes P$ type is not negligible even at $p_T \approx 0$ and $p_T \approx 0$. Configuration of J(3.1) ($\psi(3.7)$) is $c\bar{c}$ and its intrinsic spin is one, so we take $g_{pp^J} = 3r^2$, and production of J(3.1) ($\psi(3.7)$) is suppressed more than Niu particles. Detailed discussions of the new particles are found in reference 11).

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