Diversity Improvement in Multiphase Coherent PSK Communication System under One-Sided Gaussian Fading Environment

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1. INTRODUCTION

Recently, the digital communication using radio wave propagation links tends to be widely employed, and the multiphase PSK (Phase-Shift Keying) commnnication system is considered to be of more desirous performance in various systems. The radio wave propagation communication, however, unavoidably encounters with the fading phenomena which, as well-known, cause the random fluctuation on both amplitude and phase of the signal carrier. The amplitude level down and the phase variation due to fading increase, the error chances in the digital signal detection, to a large extent as compared with nonfading case.

In various techniques combatting the fading, the linear diversity combining techniques are considered to be most available and the dual diversity system (with two diversity branches) is popular for reasons of instrument and ecomy. There are three representative combining methods, i. e., 1) Selection cmbining, 2) Equal-gain combining, and 3) Maximal-ratio combining. These will not be discussed here except the mathematical modelings, since there exist many references[1] which dealt with these topics in detail.

As to fading types, most of them are the Rayleigh fadings which have been fully discussed in the previous investigations. But the recent observations[2] show the facts that the occurrences of non-Rayleigh fadings are not so rare. In this paper, we focus attention the one-sided Gaussian fading which might be deemed to be the deepest stationary fading and observed in worse propagation path (e. g., the over-sea links, etc.). The digital communication through such a fading channel will need the improvement by diversity techniques with stronger necessity. Therefore, we analytically evaluate the improvement of the detection error rates of multiphase coherent PSK signal by the predetection dual diversity systems employing three combining methods respectively, in onesided Gaussian fading channel.

2. BASIS OF ANALYSIS

Let us put the following assumptions for the analysis.

(A) The fading is very slowly tim-varying in comparison with the digital signal transmission rates.

(B) The coherent detectability is ideal, including the phase variation due to fading on the basis of assumption (A).

(C) No intersymbol-interference exists both in receiver system and in propagation path.

Under these assumptions, the phase jitters which occur the detection error in PSK system are due to the interferenc noise superimposing the signal carrer in the receiver. Let the above mentioned phase jitters be noted θ. The p. d. f. (probability density function) of θ is provided with the next formulas, ss well-known,

$$p(\theta)=\int_0^\infty f(\theta \mid r)\cdot p(r)\,dr, \tag{1}$$

where

$$f(\theta \mid r)=\frac{1}{2\pi}\exp(-r^2)\cdot\{1+\sqrt{\pi}\, r\cos\theta$$
$$\cdot\exp(r^2\cos^2\theta)\cdot[1+\mathrm{erf}(r\cos\theta)]\}. \qquad (2)^{3)}$$

The $p(r)$ is the p. d. f. of r : the faded amplitude intensity (provided with an effective value) of signal carrier normalized with respect to the effective value of noise.

If the M-phase coherent PSK signal trsnsmitted with an equal *a priori* probability is detected on the basis of the maximal-likelihood principle, the average signal rates are obtained as follows ;

$$P_e=1-\int_{-\pi/M}^{+\pi/M} p(\theta)\,d\theta. \tag{3}$$

3. SINGLE CHANNEL CASE

We start at first with a single channel case, i. e., no diversity case. In one-sided Gaussian channel, the p. d. f. of r given by

$$p(r)=\sqrt{\frac{2}{\pi\rho^2}}\cdot\exp\left(-\frac{r^2}{2\rho^2}\right), \tag{4}$$

where ρ^2 is the average CNR (Carrier-to-Noise power Ratio). Substituting Eq. (4) into Eq. (1) with Eq. (2), the p. d. f. of θ is obtained as follows;

$$p(\theta)=\frac{1}{2\pi(1+2\rho^2\sin^2\theta)}\left(\sqrt{1+2\rho^2}+\sqrt{2}\,\rho\cos\theta\right), \qquad (5)^{4)}$$

and them the integral calculation of Eq. (3) with Eq. (5) gives us

$$P_e=1-\frac{1}{\pi}\left[\mathrm{Arctan}\left(\sqrt{2}\,\rho\sin\frac{\pi}{M}\right)+\mathrm{Arctan}\left(\sqrt{1+2\rho^2}\tan\frac{\pi}{M}\right)\right]. \qquad (6)^{5)}$$

This is a special result of more general analysis.

4. DUAL DIVERSITY SYSTEM CASE

4.1 Joint p. d. f. of Dual Inputs

The joint p. d. f. of dual inputs amplitudes intensities, r_1 and r_2, in the diversity branches is described by

$$p(r_1,\ r_2) = \frac{2}{\pi\rho^2\sqrt{1-k^2}} \cdot \exp\left[-\frac{r_1^2+r_2^2}{2\rho^2(1-k^2)}\right] \cdot \cosh\left[\frac{kr_1r_2}{\rho^2(1-k^2)}\right], \qquad (7)^{6)}$$

as a special case of more general formula. It should be noted that κ^2 shows the power correlation coefficient (i. e., the correlation coefficient between r_1^2 and r_2^2), which is frequently handled rather than the ordinary correlation coefficient between r_1 and r_2, in the radiowave propagation techniques.

4.2 Selection Combining Case

The predetection selection combining method is mathematically specified as follows;

$$r = \mathrm{Max}(r_1,\ r_2)\ . \qquad (8)$$

4.2.1 p. d. f. of r

The p. d. f. of Eq. (8) is provided with

$$p(r) = \sqrt{\frac{2}{\pi\rho^2}} \cdot \exp\left(-\frac{r^2}{2\rho^2}\right)\cdot[g_s(r,\ k)+g_s(r,\ -k)], \qquad (9)^{7)}$$

where

$$g_S(r,\ k) = \mathrm{erf}\left[r\sqrt{\frac{1+k}{2\rho^2(1-k)}}\right]. \qquad (10)$$

4.2.2 p. d. f. of θ

Substituting Eq. (9) into Eq. (1) with Eq. (2), we obtain the p. d. f. of θ as follows;

$$p(\theta) = \frac{1}{\pi^2(1+2\rho^2\sin^2\theta)} \cdot [h_S(\theta,\ k)+h_S(\theta,\ -k)], \qquad (11)$$

where

$$\begin{aligned} h_S(\theta,\ k) &= \sqrt{1+2\rho^2} \cdot \mathrm{Arctan}\sqrt{\frac{1+k}{(1+2\rho^2)(1-k)}} \\ &+\rho\cos\theta \cdot \sqrt{\frac{1+k}{1+\rho^2(1-k)\sin^2\theta}} \cdot \left[\frac{\pi}{2}\right. \\ &\left.+\mathrm{Arctan}\left(\rho\cos\theta \cdot \sqrt{\frac{1-k}{1+\rho^2(1-k)\sin^2\theta}}\right)\right]. \end{aligned} \qquad (12)$$

This calculation procedure, not to be deseribed here, is very tedious, using the next formula,

$$\begin{aligned} &\int_0^\infty \exp(-\gamma)\ \mathrm{erf}(\sqrt{\alpha x})\ \mathrm{erf}(\sqrt{\beta x})\ dx \\ &= \frac{2}{\pi\gamma}\left[\sqrt{\frac{\alpha}{\alpha+\gamma}} \cdot \mathrm{Arctan}\sqrt{\frac{\beta}{\alpha+\gamma}}\right. \\ &\left.+\sqrt{\frac{\beta}{\beta+\gamma}} \cdot \mathrm{Arctan}\sqrt{\frac{\alpha}{\beta+\gamma}}\right]. \end{aligned} \qquad (13)$$

4. 2. 3 Average error rate

Erom Eq. (3), (11), and(12), the average error rate is obtained as follows;

$$P_e=1-\frac{2}{\pi^2}\left[F_S\left(\frac{\pi}{M}, k\right)+F_S\left(\frac{\pi}{M}, -k\right)\right], \tag{14}$$

where

$$F_S(\theta, k)=\text{Arctan}\sqrt{\frac{1+k}{(1+2\rho^2)(1-k)}}\cdot\text{Arctan}\left(\sqrt{1+2\rho^2}\tan\theta\right)$$
$$+\text{Arctan}\left(\rho\sin\theta\sqrt{\frac{1+k}{1+\rho^2(1-k)\sin^2\theta}}\right)$$
$$\cdot\left[\frac{\pi}{2}+\text{Arctan}\left(\rho\cos\theta\sqrt{\frac{1-k}{1+\rho^2(1-k)\sin^2\theta}}\right)\right]$$
$$+\sqrt{\frac{1-k}{1+k}}\cdot S\left[\frac{\pi}{M};\ \rho\sqrt{1-k},\ \rho\sqrt{1+k}\right], \tag{15}$$

$$S[x;\ a,\ A]=\int_0^x\frac{A\sin\theta}{\sqrt{1+a^2\sin^2\theta}}\cdot\text{Arctan}\frac{A\sin\theta}{\sqrt{1+a^2\sin^2\theta}}d\theta. \tag{16}$$

4. 3 Equal-Gain Combining Case

The predetetection equal-gain combining method is mathematically specified as follows;

$$r=(r_1+r_2)/\sqrt{2}. \tag{17}$$

As the analyses are similar to the case of **4. 2**, we show only the results.

4. 3. 1 p. d. f. of r

$$p(r)=\frac{1}{\sqrt{\pi\rho^2}}\cdot[g_{EG}(r, k)+g_{EG}(r, -k)], \tag{18}$$ [7]

where

$$g_{EG}(r, k)=\frac{1}{\sqrt{1+k}}\exp\left[-\frac{r^2}{4\rho^2(1+k)}\right]\cdot\text{erf}\left[\frac{r}{2\sqrt{\rho^2(1-k)}}\right]. \tag{19}$$

4. 3. 2 p. d. f. of θ

$$p(\theta)=\frac{1}{\pi^2}[h_{EG}(\theta, k)+h_{EG}(\theta, -k)], \tag{20}$$

where

$$h_{EG}(\theta, k)=\frac{1}{1+2\rho^2(1+k)\sin^2\theta}\left[\sqrt{1+2\rho^2(1+k)}\right.$$
$$\cdot\text{Arctan}\sqrt{\frac{1+k}{1-k+2\rho^2(1-k^2)}}$$
$$+\frac{\rho(1+k)\cos\theta}{\sqrt{1+\rho^2(1-k^2)\sin^2\theta}}\left[\frac{\pi}{2}+\text{Arctan}\left(\rho\cos\theta\right.\right.$$

$$\cdot \sqrt{\frac{1-k^2}{1+\rho^2(1-k^2)\sin^2\theta}}\Big)\Big]. \tag{21}$$

4. 3. 3 Average error rate

$$P_e = 1 - \frac{2}{\pi^2}\left[F_{EG}\left(\frac{\pi}{M}, k\right) + F_{EG}\left(\frac{\pi}{M}, -k\right)\right], \tag{22}$$

where

$$F_{EG}(\theta, k) = \mathrm{Arctan}\sqrt{\frac{1+k}{1-k+2\rho^2(1-k^2)}}$$
$$\cdot \mathrm{Arctan}(\sqrt{1+2\rho^2 (1+k)}\tan\theta)$$
$$+ \mathrm{Arctan}\frac{\rho(1+k)\sin\theta}{\sqrt{1+\rho^2(1-k^2)\sin^2\theta}} \cdot \left[\frac{\pi}{2}\right.$$
$$\left.+ \mathrm{Arctan}\left(\rho\cos\theta \cdot \sqrt{\frac{1-k^2}{1+\rho^2(1-k^2)\sin^2\theta}}\right)\right]$$
$$+ \sqrt{\frac{1-k}{1+k}} \cdot S\left[\frac{\pi}{M}, \rho\sqrt{1-k^2}, \rho(1+k)\right]. \tag{23}$$

4. 4 Maximal-Ratio Cmbining Case

The predetection maximal-ratio cmbining method is mathematically specified as follows;

$$r = \sqrt{r_1^2 + r_2^2}. \tag{24}$$

4. 4. 1 p. d. f. of r

The p. d. f. of r is given as a special case of the squared sum statistics.

$$p(r) = \frac{r}{\rho^2\sqrt{1-k^2}} \cdot \exp\left[-\frac{r^2}{2\rho^2(1-k^2)}\right]$$
$$\cdot I_0\left[\frac{kr^2}{2\rho^2(1-k^2)}\right]. \qquad (25)^{8)}$$

The way of using Eq. (25) as $p(r)$ of Eq. (1) leads into the complicated form of infinite sum for $p(\theta)$, which will have much difficulty for the numerical evaluation. Therefor, we will utilize the integral formulation of the modified Bessel function,

$$I_0(z) = \frac{1}{\pi}\int_0^\pi \exp(z\cos\varphi)\, d\varphi, \tag{26}$$

and reach the alternative form of $p(r)$:

$$p(r) = \frac{1}{\pi}\int_0^\pi \frac{r}{\rho^2\sqrt{1-k^2}}$$
$$\cdot \exp\left[-\frac{1-k\cos\varphi}{2\rho^2(1-k^2)} r^2\right] d\varphi. \tag{27}$$

4.4.2 p.d.f. of $p(\theta)$

Substituting Eq. (27) into Eq. (1) with Eq. (2) and integrating with respect to r, we obtain

$$p(\theta)=\frac{1}{2\pi^2}\int_0^{\pi}\frac{\sqrt{1-k^2}}{Q(\theta,\varphi)}\cdot\Bigg\{1+\rho\cos\theta\cdot\sqrt{\frac{2(1-k^2)}{Q(\theta,\varphi)}}\cdot\Bigg[\frac{\pi}{2}+\mathrm{Arctan}\left(\rho\cos\theta\cdot\sqrt{\frac{2(1-k^2)}{Q(\theta,\varphi)}}\right)\Bigg]\Bigg\}\,d\varphi, \tag{28}$$

where

$$Q(\theta,\varphi)=1-k\cos\varphi+2\rho^2(1-k^2)\sin^2\theta. \tag{29}$$

4.4.3. Average error rate

From Eq. (3), (28), and (29),

$$P_e=1-\frac{1}{\pi^2}\int_0^{\pi}\frac{\sqrt{1-k^2}}{1-k\cos\varphi}\Bigg\{\frac{\pi}{M}+\rho\sin\frac{\pi}{M}\cdot\sqrt{\frac{2(1-k^2)}{Q(\pi/M,\varphi)}}\Bigg[\frac{\pi}{2}+\mathrm{Arctan}\left(\rho\cos\frac{\pi}{M}\cdot\sqrt{\frac{2(1-k^2)}{Q(\pi/M,\varphi)}}\right)\Bigg]\Bigg\}\,d\varphi. \tag{30}$$

This integration on a finite interval is not so difficult to be evaluated.

5. DISCUSSION

The average errror rates P_e of M-phase coherent PSK signal given by the analysis in **3.** and **4.** can be numerically evaluated by the computer calculation with less difficulty. In this chapter the results are shown in a table and the figures and discussed in various aspects.

(i) Comparison among three combining methods

Tab. 1 Comparison of average error rates in three combining methods.

	$\rho^2=1$, $M=4$	$\rho^2=1$, $M=16$	$\rho^2=1000$, $M=4$	$\rho^2=1000$, $M=16$
$k^2=0.6$	(S) 3.31×10^{-1} (E) 3.14×10^{-1} (M) 2.91×10^{-1}	(S) 7.70×10^{-1} (E) 7.50×10^{-1} (M) 7.37×10^{-1}	(S) 9.13×10^{-4} (E) 9.13×10^{-4} (M) 7.17×10^{-4}	(S) 1.28×10^{-2} (E) 1.28×10^{-2} (M) 1.01×10^{-2}
$k^2=0.99$	(S) 4.01×10^{-1} (E) 3.29×10^{-1} (M) 3.28×10^{-1}	(S) 8.02×10^{-1} (E) 7.48×10^{-1} (M) 7.48×10^{-1}	(S) 5.28×10^{-3} (E) 5.26×10^{-3} (M) 4.23×10^{-3}	(S) 4.75×10^{-2} (E) 4.37×10^{-2} (M) 3.89×10^{-2}

(S) : Selection, (E) : Equal-gain, (M) : Maximal-ratio combining case

ρ^2 : Average carrier-to-noise ratio per one channel,

M : Number of digital phases,

k^2 : Power correlation coefficient between dual diversity intensities.

The P_e in each case of three combining methods concerned is tabulated in Tab. 1. What is evident from the table is that there is only slight difference being nearly equal to no diirference, over all ranges of the parameters, ρ^2, M, and k^2. This equality on performance suggests that the selection diversity with the merit of instrnmental simplicity is superior to both the equal-gain and the maximal-ratio one.

In the followings, we will discuss only the selection case, since the other two cases are similar to it as stated above.

(ii) P_e versus ρ^2 characteristics

In Fig. 1, some curves of P_e vs. ρ^2 of the selection case are shown. The curve of $k^2=1$ is that of on diversity, i. e., a single channel case. The CNR improvements are obtained, even if the correlation between the diversity inputs is considerably strong. It should be noted that the figure shows that the threshold effect exists at ρ^2 $(1-k^2)\sim 1$ and the remarkable improvement appear when about $\rho^2(1-k^2)>1$ (See the curves of $k^2=0.990, 0.999$). To show

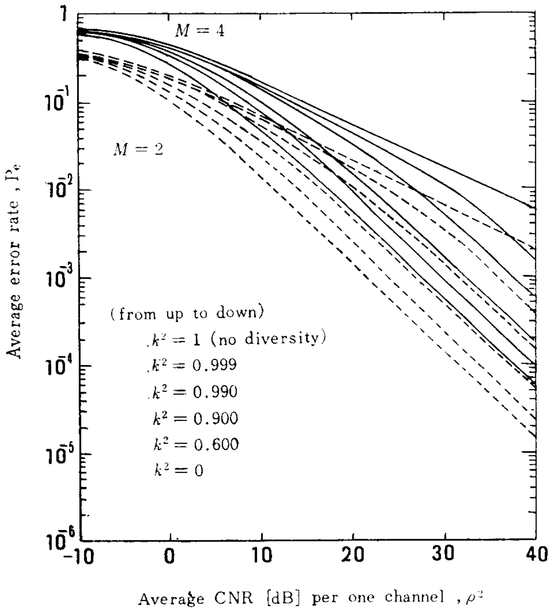

Fig. 1 Pe vs. ρ^2 in slection diversity.

this clearly, we plot the amount of improvements, i. e., the ratio $[P_e$ in no diversity or a single channel$]$, $/[P_e$ in diversity$]$ versus ρ^2 $(1-k^2)$ in dB in Fig. 2, which also shows that P_e is almost a function of only $\rho^2(1-k^2)$, not of independent ρ^2, k^2.

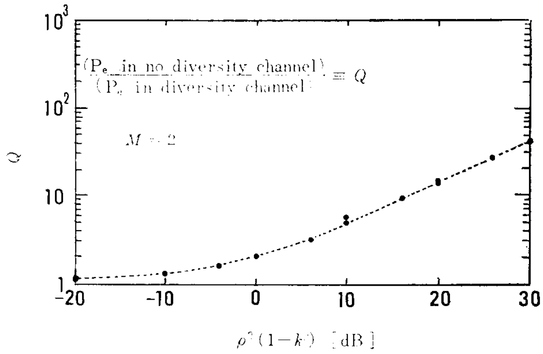

Fig. 2 Amount of improvement, Q vs. $\rho^2(1-k^2)$ in selection case.

(iii) P_e versus M characteristics

In Fig. 3, some curves of P_e vs. M are shown. the logarithmic P_e is almost linearly realated to the logarithmic M and the improvements are not exchanged by the values of M,

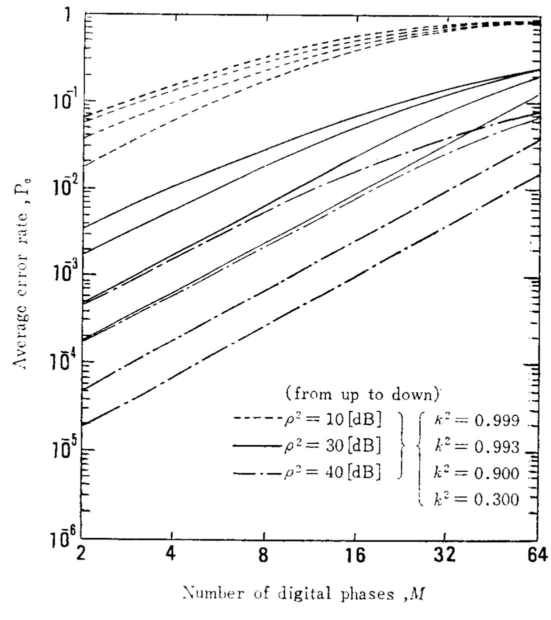

Fig. [3 Pe vs. M in selection diversity.

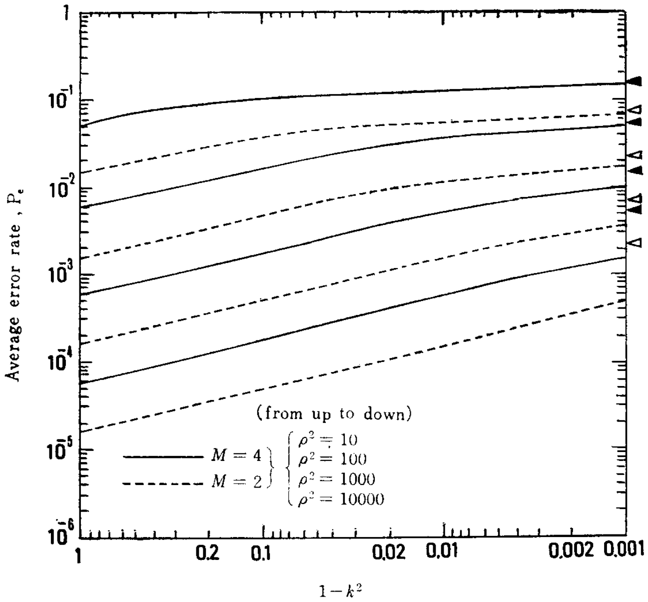

Fig. 4 Pe vs. 1-k^2 in selection case.
◀ and ◁ show Pe when $k^2=1$ (no diversity case).

when about M$\leqq$8.

(iv) P_e versus k^2 characteristics

One of the purposes of this paper is that the behavior of P_e versus the (power) correlation coefficient between diversity inputs will be explicitly discussed. In Fig. 4, some curves of P_e vs. k^2 are shown. The logarithmie P_e is linearly related to the logarithmic $1-k^2$. The effects of k^2 appear remarkably in smaller k^2 for lower ρ^2 and in larger k^2 for higher ρ^2.

6. CONCLUSION

The authors theoretically evaluated the diversity improvment as to the error rate characteristics of the multiphase coherent PSK signal, in one-sided Gaussian fading channel as a model of the stationary deepest fading channel. The conclusions are not repeated here, since they are described in 5. with various figures. Although one-sided Gaussian fading does not so frequently occur, the tendency clearified in this paper seems to be applicable to other types of fadings with some modifications.

As a problem yet to be solved in the future, more general analysis regardless of the types of fadings will be required.

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Abstract —— *In this paper, we focus attention the one-sided Gaussian fading which might be deemed to be the deepest stationary fading. The digital communication through such a fading chnnel will need the improvement by diversity techniques with stronger necessity. We analytically evaluate the improvment of the detection error rates of multiphase coherent PSK signal by the predetection dual diversity systems employing three typical combining methods in one-sided Gaussian fading channel.*

The error rates are obtained for various parameters ; the carrier-to-noise power ratio per one channel, the number of digital phases, and the power correlation coefficient beteen two diversity inputs. The results are clearly shown in a table and the figures and are discussed.