

光の直進の数量解析

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緒 言

光の直進に対する物理光学的波動論に基づき理論は古く Hugen, Fresnel に始まる事は周知の如くであるが今回我が岡山理科大学に於いて応用物理科の学生26名が卒業するに際して筆者など長期間に渉これらの専門の学者が専門的見地に立脚して光の直進に関する数量解析を試みんと欲するものである。

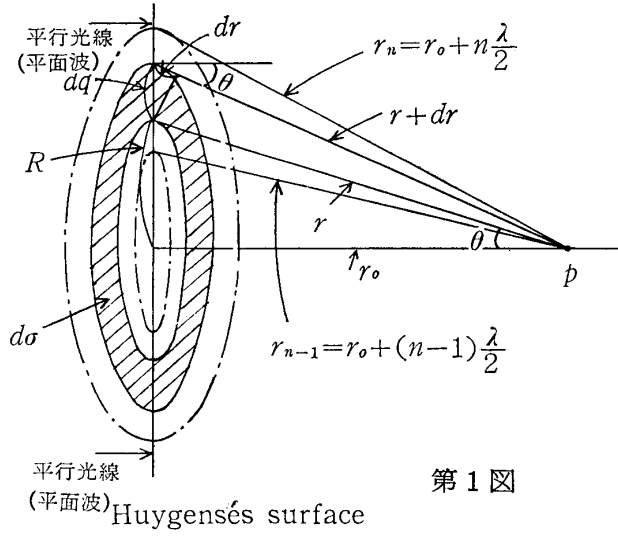
理 論

光の直進は物理光学の立場からは影の問題に帰着する事が出来る即ち影が出来ると云ふ事は光がその光路に障害物を置けば光がこないからであって光が曲ればその障害物の回りを曲って光がくる即ち影が出来なくなるわけである。従って筆者などは従来からの方法に依って光の平面波及び球面波に関して平面及び球面 Fresnel 帯に関する数学的解析をなしてその光路即ち Fresnel 帯に密着して球面及び平面の障害物を置いた時にその光波即ち optical disturbance が零となる事を下記に於いて示した。即ち下記数値計算に於いて障害物が観測点から 0.0001cm の所にある場合には第26 Fresnel 帯に於いて光波が定常状態に近づいて来るその半径は 0.004 mm であってこれで光路を遮ぎると観測点の光波は下記数値計算の如く零となる。今光でなくて波長 3m 即ち 100MC のテレビ電波に於いては obliquity factor を考へなければ第23 Fresnel 帯は半径 39m となり波長がテレビ電波の如く大きくなればとても電波の影が出来なくなると云ふわけである。

次にその距離が観測点迄 1m 位になると 1 万位迄 Fresnel 帯を計算しても少しも定常状態にならない即ち本来の物理光学では光の直進は説明出来ない、1000 Fresnel 帯に於いての計算では Fresnel 帯半径が 2.5cm である。これでは影が現実には出来る。下記数値計算に於いて Fresnel 帯番号、余弦、傾斜因子、光波、光学的合成波、光の強さ、Fresnel 帯半径、Fresnel 帯幅を示した。この中で光波が減衰してくれば良いがそうは行かない、この中で減衰してくれるのは Fresnel 帯の幅だけである。それ故ここで対象となるのは Fresnel 帯の幅だけである。この方は急速な減衰を試み上記の合成波と同様定常状態に近づく。下記数値計算に於いては $N=100 \sim 1000$ 程度の輪帯の場合 0.04mm \sim 0.012mm であり P 点には N が奇数なら $(E_{S1}/2) + (E_{SN}/2)$ 、又偶数なら $(E_{S1}/2) - (E_{SN}/2)$ の光が来る筈である。しかし障害物の縁端の部分では下記数値計算の如く Fresnel 帯の幅は非常に狭いからよほど注意しない限り N 附近の奇数偶数番目の帯を不規則に含みこれらは互に打消し合ひ結局 P 点は $E_{S1}/2$ となり第百 Fresnel 帯 $r_0=100$ の時 7.983787 mm を遮ぎれば光は P 点に達しない即ち直進をすると云ふ事となる。

I. 平 面 波

1.1 平面波 optical disturbance



第 1 図

第 1 図より

$$dr = dq \sin \theta \quad dq = \frac{dr}{\sin \theta}$$

$$R = r \sin \theta$$

微少面積を ds とす

$$\begin{aligned} ds &= 2\pi R dq = 2\pi r \sin \theta \frac{dr}{\sin \theta} \\ &= 2\pi r dr \end{aligned}$$

 r_0 ; P 点から平面 Huygens 面迄の距離 dE_s ; ds による P 点の optical disturbance E_{s_N} ; 第 N Fresel 帯に依る optical distur-

bance

但し $N = n + 1$ $n = 0, 1, 2, 3, \dots$

$$\therefore E_{s_{n+1}} = E_{s_N}$$

$$dE_s = \frac{ds}{r} q(\theta) = \frac{2\pi r dr}{r} q(\theta) = 2\pi dr q(\theta)$$

$$\therefore E_{s_{n+1}} = \int_{r_n = r_0 + n \frac{\lambda}{2}}^{r_{n+1} = r_0 + (n+1) \frac{\lambda}{2}} dE_s = 2\pi \int_{r_n}^{r_{n+1}} dr \cdot q(\theta) = \pi \lambda q(\theta)$$

故に平面波の場合に於ける optical disturbance は各 Fresnel 帯に対して obliquity factor を考へなければ相等的

$$\text{第 1 図より} \quad \cos \theta_n = \frac{r_0}{r_0 + n \frac{\lambda}{2}} = \frac{1}{1 + \frac{\lambda}{2} \frac{n}{r_0}}$$

1.2 $Q(\theta_n)$; obliquity factor $q(\theta) = \frac{1 + \cos \theta_n}{2}$

$q(\theta) = A + B \cos \theta$ とおけば obliquity factor $q(\theta)$ は $\theta = 0$ のとき 1, $\theta = \pi$ なる時 0 なる余弦函数なる故

$$\theta = 0 \text{ とおけば } A + B = 1$$

$$\theta = \pi \text{ とおけば } A - B = 0 \quad \therefore A = B \quad \text{上二式より} \quad A = B = \frac{1}{2}$$

$$\text{従って} \quad q(\theta) = \frac{1 + \cos \theta}{2}$$

$$\therefore q(\theta_n) = \frac{1 + \cos \theta_n}{2} = \frac{1}{2} \left(1 + \frac{1}{1 + \frac{\lambda}{2} \frac{n}{r_0}} \right)$$

上式は近似式でがるが厳密には

$$dE_s = 2\pi A \frac{1}{\lambda} q(\theta) \sin \left[\omega \left(t - \frac{r}{v} \right) + \frac{\pi}{2} \right] dr$$

に依らなければならない。

下記にその数値計算を表はす。

但し N ; Fresnel 帯番号 但し $N=n+1$

$\lambda(cd)$; 波長 6438.4666 \AA

$\pi=3.14159265$

$$(1) \quad \cos \theta_n = \frac{1}{1 + \frac{n}{2} \frac{\lambda}{r_0}} \quad ; \quad \text{余弦}$$

$$(2) \quad q(\theta_n) = \frac{1 + \cos \theta_n}{2} \quad ; \quad \text{傾斜因子}$$

$$(3) \quad E_{sn} = \frac{\Delta s}{r} q(\theta_n) = n \lambda q(\theta_n) \quad ; \quad \text{optical disturbance}$$

$$(4) \quad R_{s0} = E_{s0} \quad R_{s1} = E_{s0} - E_{s1} \quad R_{s2} = E_{s0} - E_{s1} + E_{s2}$$

R_{sn} ; 合成波

$$(5) \quad I_n = R_{sn}^2 \quad ; \quad \text{光の強さ}$$

$$(6) \quad \rho_n = \sqrt{n \lambda r_0} \quad ; \quad \text{Fresnel 帯半径}$$

$$(7) \quad w_n = (\sqrt{n} - \sqrt{n-1}) \sqrt{\lambda r_0} \quad ; \quad \text{Fresnel 帯の幅}$$

上式に於いて $r_0=100 \text{ cm}$ $n=0, 1, 2, 3 \dots$ を代入すれば下記数値計算が得られる。

$r_0 = 100 \text{ cm}$

N	$\cos \theta_n$	$q(\theta_n)$	E_n	R_n	I_n	ρ_n	w_n
1	1.000000	1.000000	0.00003371175	0.000033711747950	0.0000000	0.0000000
2	1.000000	1.000000	0.00003371170	0.000000000043370	0.0000000	0.0802401	0.0802401
3	0.999999	1.000000	0.00003371166	0.000033711704540	0.0000000	0.1134766	0.0332365
4	0.999999	1.000000	0.00003371162	0.000000000086790	0.0000000	0.1389799	0.0255033
5	0.999999	0.999999	0.00003371157	0.000033711661130	0.0000000	0.1604801	0.0215003
6	0.999998	0.999999	0.00003371153	0.000000000130200	0.0000000	0.1794223	0.0189421
7	0.999998	0.999999	0.00003371149	0.000033711617710	0.0000000	0.1965472	0.0171250
8	0.999998	0.999999	0.00003371144	0.000000000173610	0.0000000	0.2122953	0.0157480
9	0.999997	0.999999	0.00003371140	0.000033711574290	0.0000000	0.2269532	0.0146579
10	0.999997	0.999999	0.00003371136	0.000000000217020	0.0000000	0.2407202	0.0137670
11	0.999997	0.999998	0.00003371131	0.000033711530880	0.0000000	0.2537414	0.0130212
12	0.999996	0.999998	0.00003371127	0.000000000260440	0.0000000	0.2661262	0.0123848
13	0.999996	0.999998	0.00003371123	0.000033711487470	0.0000000	0.2779598	0.0118336
14	0.999996	0.999998	0.00003371118	0.000000000303860	0.0000000	0.2893097	0.0113499
15	0.999996	0.999998	0.00003371114	0.000033711444110	0.0000000	0.3002309	0.0109212
16	0.999995	0.999998	0.00003371110	0.000000000347270	0.0000000	0.3107685	0.0105376
17	0.999995	0.999997	0.00003371105	0.000033711400690	0.0000000	0.3209603	0.0101918
18	0.999995	0.999997	0.00003371101	0.000000000390680	0.0000000	0.3308383	0.0098780
19	0.999994	0.999997	0.00003371097	0.000033711357300	0.0000000	0.3404298	0.0095915
20	0.999994	0.999997	0.00003371092	0.000000000434120	0.0000000	0.3497284	0.0093286
21	0.999994	0.999997	0.00003371088	0.000033711313890	0.0000000	0.3588445	0.0090861
22	0.999993	0.999997	0.00003371084	0.000000000477540	0.0000000	0.3677062	0.0088617
23	0.999993	0.999996	0.00003371079	0.000033711270480	0.0000000	0.3763292	0.0086531
24	0.999993	0.999996	0.00003371075	0.000000000520930	0.0000000	0.3848179	0.0084586
25	0.999992	0.999996	0.00003371071	0.000033711227040	0.0000000	0.3930945	0.0082766
26	0.999992	0.999996	0.00003371066	0.000000000564330	0.0000000	0.4012004	0.0081059
27	0.999992	0.999996	0.00003371062	0.000033711183620	0.0000000	0.4091457	0.0079453
28	0.999991	0.999996	0.00003371058	0.000000000607700	0.0000000	0.4169397	0.0077940
29	0.999991	0.999996	0.00003371053	0.000033711140230	0.0000000	0.4245906	0.0076509
30	0.999991	0.999995	0.00003371049	0.000000000651140	0.0000000	0.4321060	0.0075155
31	0.999990	0.999995	0.00003371045	0.000033711096820	0.0000000	0.4394930	0.0073870
32	0.999990	0.999995	0.00003371040	0.000000000694530	0.0000000	0.4467278	0.0072648
33	0.999990	0.999995	0.00003371036	0.000033711053390	0.0000000	0.4539064	0.0071486
34	0.999989	0.999995	0.00003371032	0.000000000737930	0.0000000	0.4609441	0.0070377
35	0.999989	0.999995	0.00003371027	0.000033711009960	0.0000000	0.4678760	0.0069319
36	0.999989	0.999994	0.00003371023	0.000000000781320	0.0000000	0.4747067	0.0068307
37	0.999988	0.999994	0.00003371019	0.000033710966550	0.0000000	0.4814404	0.0067338
38	0.999988	0.999994	0.00003371014	0.000000000824760	0.0000000	0.4880813	0.0066409
39	0.999988	0.999994	0.00003371010	0.000033710923160	0.0000000	0.4946330	0.0065517
40	0.999988	0.999994	0.00003371005	0.000000000868200	0.0000000	0.5010991	0.0064661
41	0.999987	0.999994	0.00003371001	0.000033710879820	0.0000000	0.5074828	0.0063837
42	0.999987	0.999993	0.00003370997	0.000000000911610	0.0000000	0.5137872	0.0063044
43	0.999987	0.999993	0.00003370992	0.000033710836400	0.0000000	0.5200151	0.0062280
44	0.999986	0.999993	0.00003370988	0.000000000955020	0.0000000	0.5261694	0.0061542
45	0.999986	0.999993	0.00003370984	0.000033710793010	0.0000000	0.5322524	0.0060831
46	0.999986	0.999993	0.00003370979	0.000000000998460	0.0000000	0.5382668	0.0060143
47	0.999985	0.999993	0.00003370975	0.000033710749600	0.0000000	0.5442147	0.0059479
48	0.999985	0.999992	0.00003370971	0.000000001041880	0.0000000	0.5500982	0.0058836
49	0.999985	0.999992	0.00003370966	0.000033710706200	0.0000000	0.5559195	0.0058213
50	0.999984	0.999992	0.00003370962	0.000000001085280	0.0000000	0.5616805	0.0057610

$r_0 = 100 \text{ cm}$

N	$\cos \theta_n$	$q(\theta_n)$	E_n	R_n	I_n	ρ_n	w_n
51	0.999984	0.999992	0.00003370958	0.000033710662770	0.0000000	0.5673830	0.0057025
52	0.999984	0.999992	0.00003370953	0.000000001128670	0.0000000	0.5730288	0.0056457
53	0.999983	0.999992	0.00003370949	0.000033710619330	0.0000000	0.5786194	0.0055907
54	0.999983	0.999992	0.00003370945	0.000000001172030	0.0000000	0.5841566	0.0055372
55	0.999983	0.999991	0.00003370940	0.000033710575940	0.0000000	0.5896417	0.0054852
56	0.999982	0.999991	0.00003370936	0.000000001215470	0.0000000	0.5930762	0.0054346
57	0.999982	0.999991	0.00003370932	0.000033710532550	0.0000000	0.6004617	0.0053854
58	0.999982	0.999991	0.00003370927	0.000000001258880	0.0000000	0.6057993	0.0053375
59	0.999981	0.999991	0.00003370923	0.000033710489130	0.0000000	0.6110902	0.0052909
60	0.999981	0.999991	0.00003370919	0.000000001302290	0.0000000	0.6163357	0.0052455
61	0.999981	0.999990	0.00003370914	0.000033710445720	0.0000000	0.6215369	0.0052012
62	0.999980	0.999990	0.00003370910	0.000000001345700	0.0000000	0.6266950	0.0051581
63	0.999980	0.999990	0.00003370906	0.000033710402330	0.0000000	0.6318110	0.0051160
64	0.999980	0.999990	0.00003370901	0.000000001389140	0.0000000	0.6368858	0.0050749
65	0.999980	0.999990	0.00003370897	0.000033710358920	0.0000000	0.6419206	0.0050247
66	0.999979	0.999990	0.00003370893	0.000000001432560	0.0000000	0.6469162	0.0049956
67	0.999979	0.999989	0.00003370888	0.000033710315560	0.0000000	0.6518734	0.0049573
68	0.999979	0.999989	0.00003370884	0.000000001475950	0.0000000	0.6567933	0.0049199
69	0.999978	0.999989	0.00003370880	0.000033710272130	0.0000000	0.6616766	0.0048833
70	0.999978	0.999989	0.00003370875	0.000000001519350	0.0000000	0.6665241	0.0048475
71	0.999978	0.999989	0.00003370871	0.000033710228730	0.0000000	0.6713360	0.0048125
72	0.999977	0.999989	0.00003370867	0.000000001562770	0.0000000	0.6761149	0.0047783
73	0.999977	0.999988	0.00003370862	0.000033710185320	0.0000000	0.6808596	0.0047447
74	0.999977	0.999988	0.00003370858	0.000000001606190	0.0000000	0.6855712	0.0047119
75	0.999976	0.999988	0.00003370854	0.000033710141910	0.0000000	0.6902512	0.0046797
76	0.999976	0.999988	0.00003370849	0.000000001649580	0.0000000	0.6948994	0.0046482
77	0.999976	0.999988	0.00003370845	0.000033710098480	0.0000000	0.6995168	0.0046173
78	0.999975	0.999988	0.00003370841	0.000000001692970	0.0000000	0.7041038	0.0045870
79	0.999975	0.999988	0.00003370836	0.000033710055040	0.0000000	0.7086612	0.0045574
80	0.999975	0.999987	0.00003370832	0.000000001736310	0.0000000	0.7131894	0.0045282
81	0.999974	0.999987	0.00003370828	0.000033710011630	0.0000000	0.7176890	0.0044997
82	0.999974	0.999987	0.00003370823	0.000000001779750	0.0000000	0.7221607	0.0044716
83	0.999974	0.999987	0.00003370819	0.000033709968240	0.0000000	0.7266048	0.0044441
84	0.999973	0.999987	0.00003370815	0.000000001823150	0.0000000	0.7310219	0.0044171
85	0.999973	0.999987	0.00003370810	0.000033709924820	0.0000000	0.7354124	0.0043906
86	0.999973	0.999986	0.00003370806	0.000000001866560	0.0000000	0.7397769	0.0043645
87	0.999972	0.999986	0.00003370801	0.000033709881400	0.0000000	0.7441158	0.0043389
88	0.999972	0.999986	0.00003370797	0.000000001909960	0.0000000	0.7484296	0.0043138
89	0.999972	0.999986	0.00003370793	0.000033709838010	0.0000000	0.7527189	0.0042890
90	0.999972	0.999986	0.00003370788	0.000000001953400	0.0000000	0.7569834	0.0042647
91	0.999971	0.999986	0.00003370784	0.000033709794620	0.0000000	0.7612242	0.0042408
92	0.999971	0.999985	0.00003370780	0.000000001996830	0.0000000	0.7654415	0.0042173
93	0.999971	0.999985	0.00003370775	0.000033709751270	0.0000000	0.7696358	0.0041942
94	0.999970	0.999985	0.00003370771	0.000000002040240	0.0000000	0.7738073	0.0041715
95	0.999970	0.999985	0.00003370767	0.000033709707860	0.0000000	0.7779264	0.0041491
96	0.999970	0.999985	0.00003370762	0.000000002083650	0.0000000	0.7820835	0.0041271
97	0.999969	0.999985	0.00003370758	0.000033709664470	0.0000000	0.7861890	0.0041055
98	0.999969	0.999984	0.00003370754	0.000000002127090	0.0000000	0.7902731	0.0040841
99	0.999969	0.999984	0.00003370749	0.000033709621080	0.0000000	0.7943362	0.0040631
100	0.999968	0.999984	0.00003370745	0.000000002170520	0.0000000	0.7983787	0.0040424

$r_0 = 1 \text{ cm}$

N	$\cos \theta_n$	$q(\theta_n)$	E_n	R_n	I_n	ρ_n	w_n
1	1.000000	1.000000	0.00019263856	0.000192638559700	0.0000000	0.0000000 1
2	0.999968	0.999984	0.00019262910	0.000000009456800	0.0000000	0.0080240	0.0080240
3	0.999936	0.999968	0.00019261965	0.000192629103800	0.0000000	0.0113477	0.0033237
4	0.999904	0.999952	0.00019261019	0.000000018912300	0.0000000	0.0138980	0.0025503
5	0.999872	0.999936	0.00019260074	0.000192619649500	0.0000000	0.0160480	0.0021500
6	0.999840	0.999920	0.00019259128	0.000000028365700	0.0000000	0.0179422	0.0018942
7	0.999808	0.999904	0.00019258183	0.000192610196500	0.0000000	0.0196547	0.0017125
8	0.999776	0.999880	0.00019257238	0.000000037817600	0.0000000	0.0212295	0.0015748
9	0.999744	0.999872	0.00019256293	0.000192600745500	0.0000000	0.0226953	0.0014658
10	0.999712	0.999856	0.00019255348	0.000000047268100	0.0000000	0.0240720	0.0013767
11	0.999680	0.999840	0.00019254403	0.000192591296100	0.0000000	0.0253741	0.0013021
12	0.999648	0.999824	0.00019253458	0.000000056716700	0.0000000	0.0266126	0.0012385
13	0.999616	0.999808	0.00019252513	0.000192581848100	0.0000000	0.0277960	0.0011834
14	0.999584	0.999792	0.00019251568	0.000000066163700	0.0000000	0.0289310	0.0011350
15	0.999552	0.999776	0.00019250624	0.000192572402000	0.0000000	0.0300231	0.0010921
16	0.999520	0.999760	0.00019249679	0.000000075609300	0.0000000	0.0310768	0.0010538
17	0.999488	0.999744	0.00019248735	0.000192562957500	0.0000000	0.0320960	0.0010192
18	0.999456	0.999728	0.00019247790	0.000000085052900	0.0000000	0.0330838	0.0009878
19	0.999424	0.999712	0.00019246846	0.000192553514300	0.0000000	0.0340430	0.0009592
20	0.999392	0.999696	0.00019245902	0.000000094495000	0.0000000	0.0349758	0.0009329
21	0.999360	0.999680	0.00019244958	0.000192544073100	0.0000000	0.0358845	0.0009086
22	0.999328	0.999328	0.00019244014	0.000000103935600	0.0000000	0.0367706	0.0008862
23	0.999296	0.999648	0.00019243070	0.000192534633400	0.0000000	0.0376259	0.0008653
24	0.999265	0.999632	0.00019242126	0.000000113374700	0.0000000	0.0384818	0.0008459
25	0.999233	0.999616	0.00019241182	0.000192522195500	0.0000000	0.0393094	0.0008277
26	0.999201	0.999600	0.00019240238	0.000000122811800	0.0000000	0.0401200	0.0008106
27	0.999169	0.999584	0.00019239295	0.000192515758800	0.0000000	0.0409146	0.0007945
28	0.999137	0.999568	0.00019238351	0.000000132247300	0.0000000	0.0416940	0.0007794
29	0.999105	0.999552	0.00019237408	0.000192506324200	0.0000000	0.0424591	0.0007651
30	0.999073	0.999536	0.00019236464	0.000000141681500	0.0000000	0.0432106	0.0007515
31	0.999041	0.999520	0.00019235521	0.000192496891000	0.0000000	0.0439493	0.0007387
32	0.999009	0.999504	0.00019234578	0.000000151113700	0.0000000	0.0446758	0.0007265
33	0.998977	0.999489	0.00019233635	0.000192487459400	0.0000000	0.0453906	0.0007149
34	0.998945	0.999473	0.00019232692	0.000000160544300	0.0000000	0.0460944	0.0007038
35	0.998913	0.999457	0.00019231749	0.000192478029600	0.0000000	0.0467876	0.0006932
36	0.998881	0.999441	0.00019230806	0.000000169973700	0.0000000	0.0474707	0.0006831
37	0.998849	0.999425	0.00019229863	0.000192468601500	0.0000000	0.0481440	0.0006734
38	0.998817	0.999409	0.00019228920	0.000000179401000	0.0000000	0.0488081	0.0006641
39	0.998785	0.999393	0.00019227977	0.000192459174600	0.0000000	0.0494633	0.0006552
40	0.998754	0.999377	0.00019227035	0.000000188826800	0.0000000	0.0501099	0.0006466
41	0.998722	0.999361	0.00019226092	0.000192449749800	0.0000000	0.0507483	0.0006384
42	0.998690	0.999345	0.00019225150	0.000000198251200	0.0000000	0.0513787	0.0006304
43	0.998658	0.999329	0.00019224208	0.0001924403265000	0.0000000	0.0520015	0.0006228
44	0.998626	0.999313	0.00019223265	0.000000207673800	0.0000000	0.0526169	0.0006154
45	0.998594	0.999297	0.00019222323	0.000192430904600	0.0000000	0.0532252	0.0006083
46	0.998562	0.999281	0.00019221381	0.000000217094600	0.0000000	0.0538267	0.0006014
47	0.998530	0.999265	0.00019220439	0.000192421484300	0.0000000	0.0544215	0.0005948
48	0.998498	0.999249	0.00019219497	0.000000226513800	0.0000000	0.0550098	0.0005884
49	0.998466	0.999233	0.00019218555	0.000192412065900	0.0000000	0.0555920	0.0005821
50	0.998435	0.999217	0.00019217613	0.000000235931700	0.0000000	0.0561681	0.0005761

$r_0 = 1 \text{ cm}$

N	$\cos \theta_n$	$q(\theta_n)$	E_n	R_n	I_n	ρ_n	w_n
51	0.998403	0.999201	0.00019216672	0.000192402649200	0.0000000	0.0567383	0.0005702
52	0.998371	0.999185	0.00019215730	0.000000245347600	0.0000000	0.0573029	0.0005646
53	0.998339	0.999169	0.00019214789	0.000192393233600	0.0000000	0.0578619	0.0005591
54	0.998307	0.999153	0.00019213847	0.000000254761900	0.0000000	0.0584157	0.0005537
55	0.998275	0.999137	0.00019212906	0.000192383820100	0.0000000	0.0589642	0.0005485
56	0.998243	0.999122	0.00019211965	0.000000264174900	0.0000000	0.0595076	0.0005435
57	0.998211	0.999106	0.00019211023	0.000192374408200	0.0000000	0.0600462	0.0005385
58	0.998179	0.999090	0.00019210082	0.000000273586000	0.0000000	0.0605799	0.0005338
59	0.998147	0.999074	0.00019209141	0.000192364997600	0.0000000	0.0611090	0.0005291
60	0.998116	0.999058	0.00019208200	0.000000282995300	0.0000000	0.0616336	0.0005246
61	0.998084	0.999042	0.00019207259	0.000192355588800	0.0000000	0.0621537	0.0005201
62	0.998052	0.999026	0.00019206319	0.000000292403300	0.0000000	0.0626695	0.0005158
63	0.998020	0.999010	0.00019205378	0.000192346181800	0.0000000	0.0631811	0.0005116
64	0.997988	0.998994	0.00019204437	0.000000301809500	0.0000000	0.0636886	0.0005075
65	0.997956	0.998978	0.00019203497	0.000192336776000	0.0000000	0.0641921	0.0005035
66	0.997924	0.998962	0.00019202556	0.000000311214100	0.0000000	0.0646916	0.0004996
67	0.997892	0.998946	0.00019201616	0.0001923237372300	0.0000000	0.0651873	0.0004957
68	0.997861	0.998930	0.00019200676	0.000000320617300	0.0000000	0.0626793	0.0004920
69	0.997829	0.998914	0.00019199735	0.000192317970200	0.0000000	0.0661677	0.0004883
70	0.997797	0.998898	0.00019198795	0.000000330019000	0.0000000	0.0666524	0.0004848
71	0.997765	0.998883	0.00019197855	0.000192308569700	0.0000000	0.0671337	0.0004813
72	0.997733	0.998867	0.00019196915	0.000000339418700	0.0000000	0.0676115	0.0004778
73	0.997701	0.998851	0.00019195975	0.000192299170500	0.0000000	0.0680860	0.0004745
74	0.997669	0.998835	0.00019195035	0.000000348816800	0.0000000	0.0685571	0.0004712
75	0.997638	0.998819	0.00019194096	0.000192289773200	0.0000000	0.0690251	0.0004680
76	0.997606	0.998803	0.00019193156	0.000000358213500	0.0000000	0.0694899	0.0004648
77	0.997574	0.998787	0.00019192216	0.000192280377500	0.0000000	0.0699217	0.0004617
78	0.997542	0.998771	0.00019191277	0.000000367608300	0.0000000	0.0704104	0.0004587
79	0.997510	0.998755	0.00019190337	0.000192270983200	0.0000000	0.0708661	0.0004557
80	0.997478	0.998739	0.00019189398	0.000000377001500	0.0000000	0.0713189	0.0004528
81	0.997447	0.998723	0.00019188459	0.000192261590700	0.0000000	0.0717689	0.0004500
82	0.997415	0.998707	0.00019187520	0.000000386393400	0.0000000	0.0722161	0.0004472
83	0.997383	0.998691	0.00019186581	0.000192252199900	0.0000000	0.0726605	0.0004444
84	0.997351	0.998676	0.00019185642	0.000000395783300	0.0000000	0.0731022	0.0004417
85	0.997319	0.998660	0.00019184703	0.000192242810500	0.0000000	0.0735412	0.0004391
86	0.997287	0.998644	0.00019183764	0.000000405171600	0.0000000	0.0739777	0.0004365
87	0.997256	0.998628	0.00019182825	0.000192233422800	0.0000000	0.0744116	0.0004339
88	0.997224	0.998612	0.00019181886	0.000000414558600	0.0000000	0.0748430	0.0004314
89	0.997192	0.998596	0.00019180948	0.000192224036900	0.0000000	0.0752719	0.0004289
90	0.997160	0.998580	0.00019180009	0.000000423943700	0.0000000	0.0756983	0.0004265
91	0.997128	0.998564	0.00019179071	0.000192214652300	0.0000000	0.0761224	0.0004241
92	0.997096	0.998548	0.00019178133	0.000000433327200	0.0000000	0.0765442	0.0004217
93	0.997065	0.998532	0.00019177194	0.000192205269400	0.0000000	0.0769636	0.0004194
94	0.997033	0.998516	0.00019176256	0.000000442709200	0.0000000	0.0773807	0.0004171
95	0.997001	0.998501	0.00019175318	0.000192195888300	0.0000000	0.0777956	0.0004149
96	0.996969	0.998485	0.00019174380	0.000000452089600	0.0000000	0.0782084	0.0004127
97	0.996937	0.998469	0.00019173442	0.000192186508800	0.0000000	0.0786189	0.0004105
98	0.996906	0.998453	0.00019172504	0.000000461468300	0.0000000	0.0790273	0.0004084
99	0.996874	0.998437	0.00019171566	0.000192177130700	0.0000000	0.0794336	0.0004063
100	0.996842	0.998421	0.00019170629	0.000000470845300	0.0000000	0.0798379	0.0004042

$r_0 = 0.01\text{cm}$

N	$\cos \theta_n$	$q(\theta_n)$	E_n	R_n	I_n	ρ_n	w_n
1	1.000000	1.000000	0.00020216940	0.000202169402900	0.0000000	0.0000000
2	0.996791	0.998396	0.00020119177	0.000000972234000	0.0000000	0.0008024	0.0008024
3	0.993603	0.996801	0.00020023322	0.000201205451000	0.0000000	0.0011348	0.0003324
4	0.990435	0.995217	0.00019927745	0.000001928002100	0.0000000	0.0013898	0.0002550
5	0.987287	0.993643	0.00019832977	0.000200257769000	0.0000000	0.0016048	0.0002150
6	0.984159	0.992079	0.00019739008	0.000002867693900	0.0000000	0.0017942	0.0001894
7	0.981051	0.990525	0.00019645828	0.000199325973500	0.0000000	0.0019655	0.0001712
8	0.977962	0.988981	0.00019553429	0.000003791686900	0.0000000	0.0021230	0.0001575
9	0.974893	0.987446	0.00019461800	0.000198409691800	0.0000000	0.0022695	0.0001466
10	0.971843	0.985921	0.00019370934	0.000004700347800	0.0000000	0.0024072	0.0001377
11	0.968812	0.984406	0.00019280822	0.000197508563000	0.0000000	0.0025374	0.0001302
12	0.965800	0.982900	0.00019191453	0.000005594031900	0.0000000	0.0026613	0.0001238
13	0.962806	0.981403	0.00019102820	0.000196622236700	0.0000000	0.0027796	0.0001183
14	0.959831	0.979916	0.00019014915	0.000006473085100	0.0000000	0.0028931	0.0001135
15	0.956874	0.978437	0.00018927729	0.000195750373000	0.0000000	0.0030023	0.0001092
16	0.953936	0.976968	0.00018841253	0.000007337842000	0.0000000	0.0031077	0.0001054
17	0.951015	0.975508	0.00018755480	0.000194892641900	0.0000000	0.0032096	0.0001019
18	0.948113	0.974056	0.00018670401	0.000008188627900	0.0000000	0.0033084	0.0000988
19	0.945228	0.972614	0.00018586009	0.000194048722400	0.0000000	0.0034043	0.0000959
20	0.942360	0.971180	0.00018502296	0.000009025758700	0.0000000	0.0034976	0.0000933
21	0.939510	0.969755	0.00018419254	0.000193218303500	0.0000000	0.0035884	0.0000909
22	0.936677	0.968338	0.00018336876	0.000009849540900	0.0000000	0.0036771	0.0000886
23	0.933861	0.966931	0.00018255154	0.000192401082800	0.0000000	0.0037636	0.0000865
24	0.931062	0.965531	0.00018174081	0.000010660273100	0.0000000	0.0038482	0.0000846
25	0.928280	0.964140	0.00018093649	0.000191596766700	0.0000000	0.0039309	0.0000828
26	0.925514	0.962757	0.00018013852	0.000011458244600	0.0000000	0.0040120	0.0000811
27	0.922765	0.961382	0.00017934683	0.000190805069900	0.0000000	0.0040915	0.0000795
28	0.920031	0.960016	0.00017856133	0.000012243736500	0.0000000	0.0041694	0.0000779
29	0.917315	0.958657	0.00017778198	0.000190025714600	0.0000000	0.0042459	0.0000765
30	0.914614	0.957307	0.00017700869	0.000013017022400	0.0000000	0.0043211	0.0000752
31	0.911929	0.955964	0.00017624141	0.000189258431300	0.0000000	0.0043949	0.0000739
32	0.909259	0.954630	0.00017548006	0.000013778367800	0.0000000	0.0044676	0.0000726
33	0.906606	0.953303	0.00017472459	0.000188502958200	0.0000000	0.0045391	0.0000715
34	0.903967	0.951984	0.00017397493	0.000014528031800	0.0000000	0.0046094	0.0000704
35	0.901344	0.950672	0.00017323101	0.000187759040400	0.0000000	0.0046788	0.0000693
36	0.898736	0.949368	0.00017249278	0.000015266265400	0.0000000	0.0047471	0.0000683
37	0.896144	0.948072	0.00017176016	0.000187026430300	0.0000000	0.0048144	0.0000673
38	0.893566	0.946783	0.00017103312	0.000015993312900	0.0000000	0.0048808	0.0000664
39	0.891003	0.945501	0.00017031157	0.000186304886200	0.0000000	0.0049463	0.0000655
40	0.888454	0.944227	0.00016959547	0.000016709412200	0.0000000	0.0050110	0.0000647
41	0.885921	0.942960	0.00016888476	0.000185594174000	0.0000000	0.0050748	0.0000638
42	0.883401	0.941701	0.00016817938	0.000017414794300	0.0000000	0.0051379	0.0000630
43	0.880896	0.940448	0.00016747927	0.000184894065500	0.0000000	0.0052002	0.0000623
44	0.878405	0.939202	0.00016678438	0.000018109684800	0.0000000	0.0052617	0.0000615
45	0.875928	0.937964	0.00016609465	0.000184204338500	0.0000000	0.0053225	0.0000608
46	0.873465	0.936732	0.00016541004	0.000018794302300	0.0000000	0.0053827	0.0000601
47	0.871016	0.935508	0.00016473048	0.000183524777300	0.0000000	0.0054421	0.0000595
48	0.868580	0.934290	0.00016405592	0.000019468860400	0.0000000	0.0055010	0.0000588
49	0.866158	0.933079	0.00016338631	0.000182855171000	0.0000000	0.0055592	0.0000582
50	0.863750	0.931875	0.00016272160	0.000020133566300	0.0000000	0.0056168	0.0000576

$r_0 = 0.01\text{cm}$

N	$\cos \theta_n$	$q(\theta_n)$	E_n	R_n	I_n	ρ_n	w_n
51	0.861355	0.930677	0.00016206175	0.000182195314900	0.0000000	0.0056738	0.0000570
52	0.858973	0.929486	0.00016140669	0.000020788622000	0.0000000	0.0057303	0.0000565
53	0.856604	0.928302	0.00016075639	0.000181545009500	0.0000000	0.0057862	0.0000559
54	0.854249	0.927124	0.00016011078	0.000021434224800	0.0000000	0.0058416	0.0000554
55	0.851906	0.925953	0.00015946984	0.000180904060900	0.0000000	0.0058964	0.0000549
56	0.849576	0.924788	0.00015883349	0.000022070566400	0.0000000	0.0059508	0.0000543
57	0.847259	0.923629	0.00015820171	0.000180272279900	0.0000000	0.0060046	0.0000539
58	0.844954	0.922477	0.00015757445	0.000022697833300	0.0000000	0.0060580	0.0000534
59	0.842662	0.921331	0.00015695165	0.000179649481900	0.0000000	0.0061109	0.0000529
60	0.840382	0.920191	0.00015633327	0.000023316207500	0.0000000	0.0061634	0.0000525
61	0.838115	0.919057	0.00015571928	0.000179035487300	0.0000000	0.0062154	0.0000520
62	0.835859	0.917930	0.00015510962	0.000023925866000	0.0000000	0.0062670	0.0000516
63	0.833616	0.916808	0.00015450426	0.000178430121200	0.0000000	0.0063181	0.0000512
64	0.831385	0.915693	0.00015390314	0.000024526982100	0.0000000	0.0063689	0.0000507
65	0.829166	0.914583	0.00015330623	0.000177833213100	0.0000000	0.0064192	0.0000503
66	0.826959	0.913479	0.00015271349	0.000025119724000	0.0000000	0.0064692	0.0000500
67	0.824763	0.912382	0.00015212487	0.000177244596900	0.0000000	0.0065187	0.0000496
68	0.822579	0.911289	0.00015154034	0.000025704255700	0.0000000	0.0065679	0.0000492
69	0.820406	0.910203	0.00015095985	0.000176664110100	0.0000000	0.0066168	0.0000488
70	0.818245	0.909123	0.00015038337	0.000026280737300	0.0000000	0.0066652	0.0000485
71	0.816096	0.908048	0.00014981086	0.000176091594700	0.0000000	0.0067124	0.0000481
72	0.813957	0.906979	0.00014924227	0.000026849324700	0.0000000	0.0067611	0.0000478
73	0.811830	0.905915	0.00014867757	0.000175526896900	0.0000000	0.0068086	0.0000474
74	0.809714	0.904857	0.00014811673	0.000027410170600	0.0000000	0.0068257	0.0000471
75	0.807609	0.903804	0.00014755970	0.000174969866200	0.0000000	0.0069025	0.0000468
76	0.805514	0.902757	0.00014700644	0.000027963423000	0.0000000	0.0069490	0.0000465
77	0.803431	0.901716	0.00014645693	0.000174420356100	0.0000000	0.0069952	0.0000462
78	0.801358	0.900679	0.00014591113	0.000028509226800	0.0000000	0.0070410	0.0000459
79	0.799296	0.899648	0.00014536900	0.000173878223400	0.0000000	0.0070866	0.0000456
80	0.797245	0.898623	0.00014483050	0.000029047723400	0.0000000	0.0071219	0.0000453
81	0.795204	0.897602	0.00014429561	0.000173343328600	0.0000000	0.0071769	0.0000450
82	0.793174	0.896587	0.00014376428	0.000029579050500	0.0000000	0.0072216	0.0000447
83	0.791153	0.895577	0.00014323649	0.000172815535600	0.0000000	0.0072660	0.0000444
84	0.789144	0.894572	0.00014271219	0.000030103342800	0.0000000	0.0073102	0.0000442
85	0.787144	0.893572	0.00014219137	0.000172294711500	0.0000000	0.0073541	0.0000439
86	0.785154	0.892577	0.00014167398	0.000030620731500	0.0000000	0.0073978	0.0000436
87	0.783175	0.891587	0.00014116000	0.000171780726500	0.0000000	0.0074412	0.0000434
88	0.781205	0.890603	0.00014064938	0.000031131344800	0.0000000	0.0074843	0.0000431
89	0.779245	0.889623	0.00014014211	0.000171273453900	0.0000000	0.0075272	0.0000429
90	0.777296	0.888648	0.00013963815	0.000031635307800	0.0000000	0.0075698	0.0000426
91	0.775355	0.887678	0.00013913746	0.000170772770000	0.0000000	0.0076122	0.0000424
92	0.773425	0.886712	0.00013864003	0.000032132742500	0.0000000	0.0076544	0.0000422
93	0.771504	0.885752	0.00013814581	0.000170278554300	0.0000000	0.0076964	0.0000419
94	0.769593	0.884796	0.00013765479	0.000032623768500	0.0000000	0.0077381	0.0000417
95	0.767691	0.883845	0.00013716692	0.000169790688700	0.0000000	0.0077796	0.0000415
96	0.765798	0.882899	0.00013668219	0.000033108502300	0.0000000	0.0078208	0.0000413
97	0.763915	0.881957	0.00013620056	0.000169309058300	0.0000000	0.0078619	0.0000411
98	0.762041	0.881020	0.00013572200	0.000033587057600	0.0000000	0.0079027	0.0000408
99	0.760176	0.880088	0.00013524649	0.000168833550400	0.0000000	0.0079434	0.0000406
100	0.758320	0.879160	0.00013477400	0.000034059545600	0.0000000	0.0079838	0.0000404

$r_0 = 0.0001 \text{ cm}$

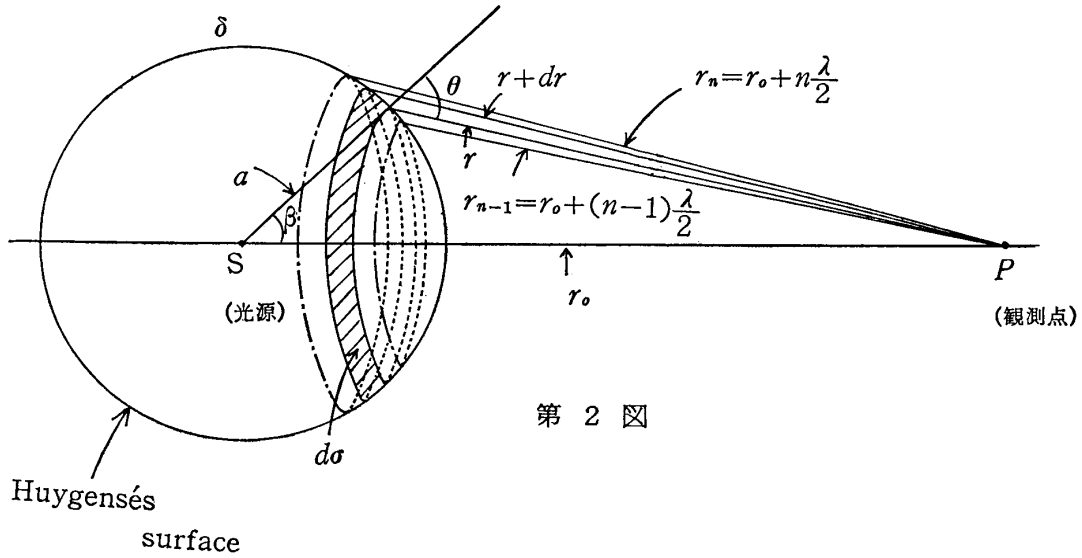
N	$\cos \theta_n$	$q(\theta_n)$	E_n	R_n	I_n	ρ_n	w_n
1	1.000000	1.000000	0.00020226948	0.000202269476300	0.0000000	0.0000000
2	0.756473	0.878237	0.00013438021	0.000067889263900	0.0000000	0.0000802	0.0000802
3	0.608329	0.804165	0.00009894945	0.000166838711000	0.0000000	0.0001135	0.0000332
4	0.508706	0.754353	0.00007761963	0.000089219079040	0.0000000	0.0001390	0.0000255
5	0.437121	0.718561	0.00006353237	0.000152751447500	0.0000000	0.0001605	0.0000215
6	0.383198	0.691599	0.00005360519	0.000099146252690	0.0000000	0.0001794	0.0000189
7	0.341118	0.670559	0.00004626688	0.000145413128400	0.0000000	0.0001965	0.0000171
8	0.307365	0.653682	0.00004063964	0.000104773484400	0.0000000	0.0002123	0.0000157
9	0.279690	0.639845	0.00003619768	0.000140971163700	0.0000000	0.0002270	0.0000147
10	0.256587	0.628294	0.00003260818	0.000108362981800	0.0000000	0.0002407	0.0000138
11	0.237010	0.618505	0.00002965094	0.000138013921400	0.0000000	0.0002537	0.0000130
12	0.220208	0.610104	0.00002717479	0.000110839128400	0.0000000	0.0002661	0.0000124
13	0.205631	0.602815	0.00002507274	0.000135911864600	0.0000000	0.0002780	0.0000118
14	0.192864	0.596432	0.00002326701	0.000112644855600	0.0000000	0.0002893	0.0000113
15	0.181589	0.590795	0.00002169981	0.000134344668500	0.0000000	0.0003002	0.0000109
16	0.171560	0.585780	0.00002032734	0.000114017333400	0.0000000	0.0003108	0.0000105
17	0.162581	0.581291	0.00001911579	0.000133133119700	0.0000000	0.0003210	0.0000102
18	0.154495	0.577248	0.00001803871	0.000115094412100	0.0000000	0.0003308	0.0000099
19	0.147175	0.573588	0.00001707510	0.000132169507900	0.0000000	0.0003404	0.0000096
20	0.140518	0.570259	0.00001620808	0.000115961432700	0.0000000	0.0003498	0.0000093
21	0.134436	0.567218	0.00001542394	0.000131385368200	0.0000000	0.0003588	0.0000091
22	0.128859	0.564430	0.00001471143	0.000116673940100	0.0000000	0.0003677	0.0000089
23	0.123727	0.561863	0.00001406124	0.000130735178800	0.0000000	0.0003764	0.0000087
24	0.118988	0.559494	0.00001346559	0.000117269586700	0.0000000	0.0003848	0.0000085
25	0.114598	0.557299	0.00001291795	0.000130187533800	0.0000000	0.0003931	0.0000083
26	0.110521	0.555260	0.00001241276	0.000117774770700	0.0000000	0.0004012	0.0000081
27	0.106723	0.553362	0.00001194532	0.000129720086700	0.0000000	0.0004091	0.0000079
28	0.103179	0.551589	0.00001151155	0.000118208532400	0.0000000	0.0004169	0.0000078
29	0.099862	0.549931	0.00001110798	0.000129316515700	0.0000000	0.0004246	0.0000077
30	0.096751	0.548376	0.00001073157	0.000118584941800	0.0000000	0.0004321	0.0000075
31	0.093829	0.546914	0.00001037969	0.000128964628600	0.0000000	0.0004395	0.0000074
32	0.091078	0.545539	0.00001005001	0.000118914615400	0.0000000	0.0004468	0.0000073
33	0.088483	0.544242	0.00000974052	0.000128655138700	0.0000000	0.0004539	0.0000071
34	0.086033	0.543016	0.00000944943	0.000119205711400	0.0000000	0.0004609	0.0000070
35	0.083714	0.541857	0.00000917514	0.000128380850600	0.0000000	0.0004679	0.0000069
36	0.081517	0.540759	0.00000891625	0.000119464600100	0.0000000	0.0004747	0.0000068
37	0.079433	0.539716	0.00000867150	0.000128136104500	0.0000000	0.0004814	0.0000067
38	0.077452	0.538726	0.00000843978	0.000119696326900	0.0000000	0.0004881	0.0000066
39	0.075568	0.537784	0.00000822006	0.000127916388500	0.0000000	0.0004946	0.0000066
40	0.073773	0.536887	0.00000801145	0.000119904938800	0.0000000	0.0005011	0.0000065
41	0.072062	0.536031	0.00000781312	0.000127718062300	0.0000000	0.0005075	0.0000064
42	0.070428	0.535214	0.00000762434	0.000120093719000	0.0000000	0.0005138	0.0000063
43	0.068867	0.534433	0.00000744444	0.000127538157000	0.0000000	0.0005200	0.0000062
44	0.067373	0.533687	0.00000727280	0.000120265358800	0.0000000	0.0005262	0.0000062
45	0.065943	0.532971	0.00000710887	0.000127374227400	0.0000000	0.0005323	0.0000061
46	0.064572	0.532286	0.00000695214	0.000120422084700	0.0000000	0.0005383	0.0000060
47	0.063257	0.531629	0.00000680216	0.000127224241700	0.0000000	0.0005442	0.0000059
48	0.061995	0.530997	0.00000665849	0.000120565754500	0.0000000	0.0005501	0.0000059
49	0.060782	0.530391	0.00000652074	0.000127086497800	0.0000000	0.0005559	0.0000058
50	0.059615	0.529808	0.00000638857	0.000120697930400	0.0000000	0.0005617	0.0000058

$r_0 = 0.0001 \text{ cm}$

N	$\cos \theta_n$	$q(\theta_n)$	E_n	R_n	I_n	ρ_n	w_n
51	0.058493	0.529246	0.00000626163	0.000126959559300	0.0000000	0.0005674	0.0000057
52	0.057412	0.528706	0.00000613962	0.000120819935600	0.0000000	0.0005730	0.0000056
53	0.056370	0.528185	0.00000602227	0.000126842205600	0.0000000	0.0005786	0.0000056
54	0.055365	0.527683	0.00000590931	0.000120932897800	0.0000000	0.0005842	0.0000055
55	0.054396	0.527198	0.00000580050	0.000126733392900	0.0000000	0.0005896	0.0000055
56	0.053459	0.526730	0.00000569561	0.000121037784600	0.0000000	0.0005951	0.0000054
57	0.052555	0.526277	0.00000559444	0.000126632223400	0.0000000	0.0006005	0.0000054
58	0.051681	0.525840	0.00000549679	0.000121135430300	0.0000000	0.0006058	0.0000053
59	0.050835	0.525417	0.00000540249	0.000126537920600	0.0000000	0.0006111	0.0000053
60	0.050016	0.525008	0.00000531136	0.000121226558300	0.0000000	0.0006163	0.0000052
61	0.049224	0.524612	0.00000522325	0.000126449809600	0.0000000	0.0006215	0.0000052
62	0.048456	0.524228	0.00000513801	0.000121311799100	0.0000000	0.0006267	0.0000052
63	0.047712	0.523856	0.00000505550	0.000126367301000	0.0000000	0.0006318	0.0000051
64	0.046990	0.523495	0.00000497560	0.000121391704300	0.0000000	0.0006369	0.0000051
65	0.046290	0.523145	0.00000489817	0.000126289877800	0.0000000	0.0006419	0.0000050
66	0.045610	0.522805	0.00000482312	0.000121466758800	0.0000000	0.0006469	0.0000050
67	0.044950	0.522475	0.00000475033	0.000126217084600	0.0000000	0.0006519	0.0000050
68	0.044309	0.522154	0.00000467969	0.000121537390800	0.0000000	0.0006568	0.0000049
69	0.043686	0.521843	0.00000461113	0.000126148518800	0.0000000	0.0006617	0.0000049
70	0.043080	0.521540	0.00000454454	0.000121603979300	0.0000000	0.0006665	0.0000048
71	0.042491	0.521245	0.00000447984	0.000126083822900	0.0000000	0.0006713	0.0000048
72	0.041917	0.520959	0.00000441696	0.000121666861400	0.0000000	0.0006761	0.0000048
73	0.041359	0.520680	0.00000435582	0.000126022678900	0.0000000	0.0006809	0.0000047
74	0.040816	0.520408	0.00000429634	0.000121726337900	0.0000000	0.0006856	0.0000047
75	0.040286	0.520143	0.00000423846	0.000125964802400	0.0000000	0.0006903	0.0000047
76	0.039771	0.519885	0.00000418212	0.000121782677800	0.0000000	0.0006949	0.0000046
77	0.039268	0.519634	0.00000412726	0.000125909938600	0.0000000	0.0006995	0.0000046
78	0.038778	0.419389	0.00000407382	0.000121836122400	0.0000000	0.0007041	0.0000046
79	0.038299	0.519150	0.00000402174	0.000125857858500	0.0000000	0.0007087	0.0000046
80	0.037833	0.518916	0.00000397097	0.000121886889100	0.0000000	0.0007132	0.0000045
81	0.037378	0.518689	0.00000392147	0.000125808355700	0.0000000	0.0007177	0.0000045
82	0.036933	0.518467	0.00000387318	0.000121935174100	0.0000000	0.0007222	0.0000045
83	0.036499	0.518250	0.00000382607	0.000125761243700	0.0000000	0.0007266	0.0000044
84	0.036075	0.518038	0.00000378009	0.000121981154800	0.0000000	0.0007310	0.0000044
85	0.035661	0.517831	0.00000373520	0.000125716353700	0.0000000	0.0007354	0.0000044
86	0.035257	0.517628	0.00000369136	0.000122024992200	0.0000000	0.0007398	0.0000044
87	0.034861	0.517430	0.00000364854	0.000125673532200	0.0000000	0.0007441	0.0000043
88	0.034474	0.517237	0.00000360670	0.000122066832500	0.0000000	0.0007484	0.0000043
89	0.034096	0.517048	0.00000356581	0.000125632639500	0.0000000	0.0007527	0.0000043
90	0.033725	0.516863	0.00000352583	0.000122106809100	0.0000000	0.0007570	0.0000043
91	0.033363	0.516682	0.00000348674	0.000125593548300	0.0000000	0.0007612	0.0000042
92	0.033009	0.516504	0.00000344850	0.000122145043600	0.0000000	0.0007654	0.0000042
93	0.032662	0.516331	0.00000341110	0.000125556142300	0.0000000	0.0007696	0.0000042
94	0.032322	0.516161	0.00000337449	0.000122181647400	0.0000000	0.0007738	0.0000042
95	0.031989	0.515994	0.00000333867	0.000125520314900	0.0000000	0.0007780	0.0000041
96	0.031663	0.515831	0.00000330359	0.000122216722600	0.0000000	0.0007821	0.0000041
97	0.031343	0.515672	0.00000326925	0.000125485968300	0.0000000	0.0007862	0.0000041
98	0.031030	0.515515	0.00000323561	0.000122250362900	0.0000000	0.0007903	0.0000041
99	0.030723	0.515362	0.00000320265	0.000125453012600	0.0000000	0.0007943	0.0000041
100	0.030422	0.515211	0.00000317036	0.000122282654400	0.0000000	0.0007984	0.0000040

II. 球面波

2.1 球面波 optical disturbance



第 2 図

S; 光源

a; 球半径

 r_0 ; P から球面 Huygens 面迄の距離

ds; 微少面積

$$ds = \frac{2\pi ar}{a+r_0} dr$$

$$dE_S = \frac{ds}{r} q(\theta) = \frac{2\pi a}{a+r_0} dr q(\theta)$$

$$\therefore E_{S_{n+1}} = \int_{r_n}^{r_{n+1}} dE_S = \frac{2\pi a}{a+r_0} \int_{r_n=r_0+n\frac{\lambda}{2}}^{r_{n+1}=r_0+(n+1)\frac{\lambda}{2}} dr q(\theta) = \frac{\pi \lambda a}{a+r_0} q(\theta)$$

但し $N=n+1$
 $n=0, 1, 2, 3, \dots$
 $E_{S_N} = E_{S_{n+1}}$

 dE_S ; ds による P 点の optical disturbance E_{S_N} ; 第 N Fresnel 帯に依る optical disturbance

第 2 図に於いて余弦第二定理に依り

$$(a+r_0)^2 = a^2 + r_n^2 + 2ar_n \cos \theta_n$$

次に $r_n = r_0 + n\frac{\lambda}{2}$ を代入

$$(a+r_0)^2 = a^2 + \left(r_0 + n\frac{\lambda}{2}\right)^2 + 2a\left(r_0 + n\frac{\lambda}{2}\right) \cos \theta_n$$

$$\therefore \cos \theta_n = \frac{1 - \frac{n\lambda}{2a}}{1 + \frac{\lambda}{2} \frac{n}{r_0}}$$

 $a \rightarrow \infty$ とすれば平面波の場合と一致する

2.2 obliquity factor $q(\theta_n)$

$$q(\theta_n) = \frac{1 + \cos \theta_n}{2} = \frac{1}{2} \left(1 + \frac{1 - \frac{n\lambda}{2a}}{1 + \frac{\lambda}{2} \frac{n}{r_0}} \right)$$

2.3 Fresnel 帯正射影

第2図に於いて余弦定理により

$$r_n^2 = \left(r_1 + n \frac{\lambda}{2} \right)^2 = a^2 + (a + r_0)^2 + 2a(a + r_0) \cos \beta_n$$

上式より $\cos \beta_n = 1 - \frac{r_0 n \lambda}{2a(a + r_0)}$

$$\cos^2 \beta_n = 1 - 2 \frac{n \lambda r_0}{2a(a + r_0)} + \left[\frac{r_0 n \lambda}{2a(a + r_0)} \right]^2$$

λ が小さい故上式第3項二次式を消去すれば

$$\cos^2 \beta_n = 1 - \frac{n \lambda r_0}{a(a + r_0)}$$

$$\sin^2 \beta_n = 1 - \cos^2 \beta_n = 1 - 1 + \frac{n \lambda r_0}{a(a + r_0)}$$

次に第 n Fresnel 帯正射影半径を R_n とすれば

$$R_n^2 = a^2 \sin^2 \beta_n = \frac{an \lambda r_0}{a + r_0} \quad \therefore R_n = \sqrt{\frac{an \lambda r_0}{a + r_0}}$$

第 n Fresnel 帯幅を w_n とすれば

$$w_n = (\sqrt{n} - \sqrt{n-1}) \sqrt{\frac{a \lambda r_0}{a + r_0}}$$

下記の数値計算を表はす

但し $\lambda = 6433.4696 \text{ \AA}$ N ; Fresnel 帯番号 π ; 3.14159265

(1) $\cos \theta_n$; 余弦

(2) $q(\theta_n) = \frac{1 + \cos \theta_n}{2}$; obliquity factor

(3) $E_{Sn} = \int \frac{ds}{r} q(\theta_n) = \frac{\pi \lambda a}{a + r_0} q(\theta)$; optical disturbance

(4) $R_{S0} = E_{S0}$ $R_{S1} = E_{S0} - E_{S1}$ $R_{S2} = E_{S1} - E_{S1} + E_{S2}$ R_S : 合成波

(5) $I_n = R_{Sn}^2$; 光の強さ

(6) $p_n = \sqrt{\frac{an \lambda r_0}{a + r_0}}$; Fresnel 帯半径

(7) $w_n = (\sqrt{n} - \sqrt{n-1}) \sqrt{\frac{a \lambda r_0}{a + r_0}}$; Fresnel 帯の幅

本数値計算は岡山理科大学応用数学科の高速交電子計算機によるもので一挙にして数10万に及ぶ計算をして下さった同研究室木村宏氏、石井まき子氏に深甚なる謝意を表明する次第である。

$r_0=100$ cm

N	$\cos \theta_n$	$q(\theta_n)$	E_{sn}	R_{so}	I_n	ρ_n	w_n
1	1.000000	1.000000	0.000034	0.0000337117	0.0000000011	0.032757873
2	0.999998	0.999999	0.000034	0.0000000000	0.0000000000	0.046326629	0.032757873
3	0.999996	0.999998	0.000034	0.0000337117	0.0000000011	0.056738301	0.013568755
4	0.999994	0.999997	0.000034	0.0000000001	0.0000000000	0.065515747	0.010411672
5	0.999992	0.999996	0.000034	0.0000337117	0.0000000011	0.073248832	0.008777446
6	0.999990	0.999995	0.000034	0.0000000001	0.0000000000	0.080240075	0.007733085
7	0.999988	0.999994	0.000034	0.0000337117	0.0000000011	0.086669186	0.006991243
8	0.999986	0.999993	0.000034	0.0000000001	0.0000000000	0.092653258	0.006429112
9	0.999985	0.999992	0.000034	0.0000337116	0.0000000011	0.098273620	0.005984071
10	0.999983	0.999991	0.000034	0.0000000002	0.0000000000	0.103589491	0.005620363
11	0.999981	0.999990	0.000034	0.0000337116	0.0000000011	0.108645575	0.005315871
12	0.999979	0.999989	0.000034	0.0000000002	0.0000000000	0.113476602	0.005056084
13	0.999977	0.999988	0.000034	0.0000337116	0.0000000011	0.118110192	0.004831027
14	0.999975	0.999987	0.000034	0.0000000002	0.0000000000	0.122568739	0.004633590
15	0.999973	0.999986	0.000034	0.0000337115	0.0000000011	0.126870698	0.004458547
16	0.999971	0.999986	0.000034	0.0000000003	0.0000000000	0.131031493	0.004301959
17	0.999969	0.999985	0.000034	0.0000337115	0.0000000011	0.135064172	0.004160795
18	0.999967	0.999984	0.000034	0.0000000003	0.0000000000	0.138979886	0.004032678
19	0.999965	0.999983	0.000034	0.0000337115	0.0000000011	0.142788260	0.003915714
20	0.999963	0.999982	0.000034	0.0000000003	0.0000000000	0.146497663	0.003808373
21	0.999961	0.999981	0.000034	0.0000337114	0.0000000011	0.150115434	0.003709404
22	0.999959	0.999980	0.000034	0.0000000004	0.0000000000	0.153648045	0.003617771
23	0.999958	0.999979	0.000034	0.0000337114	0.0000000011	0.157101242	0.003532611
24	0.999956	0.999978	0.000034	0.0000000004	0.0000000000	0.160480150	0.003453196
25	0.999954	0.999977	0.000034	0.0000337114	0.0000000011	0.163789367	0.003378908
26	0.999952	0.999976	0.000034	0.0000000004	0.0000000000	0.167033035	0.003309217
27	0.999950	0.999975	0.000034	0.0000337113	0.0000000011	0.170214903	0.003243669
28	0.999948	0.999974	0.000034	0.0000000005	0.0000000000	0.173338373	0.003181868
29	0.999946	0.999973	0.000034	0.0000337113	0.0000000011	0.176406547	0.003123470
30	0.999944	0.999972	0.000034	0.0000000005	0.0000000000	0.179422262	0.003068174
31	0.999942	0.999971	0.000034	0.0000337113	0.0000000011	0.182388120	0.003015715
32	0.999940	0.999970	0.000034	0.0000000005	0.0000000000	0.185306515	0.002965858
33	0.999938	0.999969	0.000034	0.0000337112	0.0000000011	0.188179656	0.002918395
34	0.999936	0.999968	0.000034	0.0000000006	0.0000000000	0.191009584	0.002873141
35	0.999934	0.999967	0.000034	0.0000337112	0.0000000011	0.193798192	0.002829928
36	0.999932	0.999966	0.000034	0.0000000006	0.0000000000	0.196547240	0.002788609
37	0.999930	0.999965	0.000034	0.0000337112	0.0000000011	0.199258365	0.002749048
38	0.999929	0.999964	0.000034	0.0000000006	0.0000000000	0.201933093	0.002711124
39	0.999927	0.999963	0.000034	0.0000337111	0.0000000011	0.204572854	0.002674729
40	0.999925	0.999962	0.000034	0.0000000007	0.0000000000	0.207178982	0.002639760
41	0.999923	0.999961	0.000034	0.0000337111	0.0000000011	0.209752733	0.002606129
42	0.999921	0.999960	0.000034	0.0000000007	0.0000000000	0.212295283	0.002573751
43	0.999919	0.999959	0.000034	0.0000337111	0.0000000011	0.214807741	0.002542550
44	0.999917	0.999958	0.000034	0.0000000007	0.0000000000	0.217291150	0.002512458
45	0.999915	0.999958	0.000034	0.0000337110	0.0000000011	0.219746495	0.002483409
46	0.999913	0.999957	0.000034	0.0000000007	0.0000000000	0.222174707	0.002455345
47	0.999911	0.999956	0.000034	0.0000337110	0.0000000011	0.224576665	0.002428212
48	0.999909	0.999955	0.000034	0.0000000008	0.0000000000	0.226953204	0.002401959
49	0.999907	0.999954	0.000034	0.0000337110	0.0000000011	0.229305113	0.002376539
50	0.999905	0.999953	0.000034	0.0000000008	0.0000000000	0.231633144	0.002351909

$r_0 = 100 \text{ cm}$

N	$\cos \theta_n$	$q(\theta_n)$	E_{Sn}	R_{SO}	I_n	ρ_n	w_n
51	0.999903	0.999952	0.000034	0.0000337109	0.0000000011	0.233938008	0.002328030
52	0.999901	0.999951	0.000034	0.0000000008	0.0000000000	0.236220384	0.002304864
53	0.999900	0.999950	0.000034	0.0000337109	0.0000000011	0.238480918	0.002282376
54	0.999898	0.999949	0.000034	0.0000000009	0.0000000000	0.240720224	0.002260534
55	0.999896	0.999948	0.000034	0.0000337109	0.0000000011	0.242938891	0.002239307
56	0.999894	0.999947	0.000034	0.0000000009	0.0000000000	0.245137478	0.002218666
57	0.999892	0.999946	0.000034	0.0000337108	0.0000000011	0.247316520	0.002198587
58	0.999890	0.999945	0.000034	0.0000000009	0.0000000000	0.249476531	0.002179043
59	0.999888	0.999944	0.000034	0.0000337108	0.0000000011	0.251618000	0.002160011
60	0.999886	0.999943	0.000034	0.0000000010	0.0000000000	0.253741396	0.002141469
61	0.999884	0.999942	0.000034	0.0000337108	0.0000000011	0.255847170	0.002123396
62	0.999882	0.999941	0.000034	0.0000000010	0.0000000000	0.257935753	0.002105774
63	0.999880	0.999940	0.000034	0.0000337107	0.0000000011	0.260007559	0.002088583
64	0.999878	0.999939	0.000034	0.0000000010	0.0000000000	0.262062987	0.002071806
65	0.999876	0.999938	0.000034	0.0000337107	0.0000000011	0.264102418	0.002055428
66	0.999874	0.999937	0.000034	0.0000000011	0.0000000000	0.266126221	0.002039431
67	0.999873	0.999936	0.000034	0.0000337107	0.0000000011	0.268134749	0.002023803
68	0.999871	0.999935	0.000034	0.0000000011	0.0000000000	0.270128344	0.002008528
69	0.999869	0.999934	0.000034	0.0000337106	0.0000000011	0.272107332	0.001993594
70	0.999867	0.999933	0.000034	0.0000000011	0.0000000000	0.274072032	0.001978989
71	0.999865	0.999932	0.000034	0.0000337106	0.0000000011	0.276022747	0.001964699
72	0.999863	0.999931	0.000034	0.0000000012	0.0000000000	0.277929773	0.001950715
73	0.999861	0.999930	0.000034	0.0000337106	0.0000000011	0.279883393	0.001937025
74	0.999859	0.999930	0.000034	0.0000000012	0.0000000000	0.281793882	0.001923620
75	0.999857	0.999929	0.000034	0.0000337105	0.0000000011	0.283691505	0.001910489
76	0.999855	0.999928	0.000034	0.0000000012	0.0000000000	0.285576519	0.001897623
77	0.999853	0.999927	0.000034	0.0000337105	0.0000000011	0.287449172	0.001885014
78	0.999851	0.999926	0.000034	0.0000000013	0.0000000000	0.289309704	0.001872653
79	0.999849	0.999925	0.000034	0.0000337105	0.0000000011	0.291158347	0.001860532
80	0.999847	0.999924	0.000034	0.0000000013	0.0000000000	0.292995326	0.001848643
81	0.999845	0.999923	0.000034	0.0000337104	0.0000000011	0.294820860	0.001836979
82	0.999844	0.999922	0.000034	0.0000000013	0.0000000000	0.296635160	0.001825534
83	0.999842	0.999921	0.000034	0.0000337104	0.0000000011	0.298438429	0.001814299
84	0.999840	0.999920	0.000034	0.0000000014	0.0000000000	0.300230868	0.001803270
85	0.999838	0.999919	0.000034	0.0000337104	0.0000000011	0.302012670	0.001792439
86	0.999836	0.999918	0.000034	0.0000000014	0.0000000000	0.303784020	0.001781801
87	0.999834	0.999917	0.000034	0.0000337103	0.0000000011	0.305545102	0.001771350
88	0.999832	0.999916	0.000034	0.0000000014	0.0000000000	0.307296091	0.001761082
89	0.999830	0.999915	0.000034	0.0000337103	0.0000000011	0.309037159	0.001750989
90	0.999828	0.999914	0.000034	0.0000000015	0.0000000000	0.310768473	0.001741068
91	0.999826	0.999913	0.000034	0.0000337103	0.0000000011	0.312490195	0.001731314
92	0.999824	0.999912	0.000034	0.0000000015	0.0000000000	0.314202483	0.001721722
93	0.999822	0.999911	0.000034	0.0000337103	0.0000000011	0.315905490	0.001712288
94	0.999820	0.999910	0.000034	0.0000000015	0.0000000000	0.317599366	0.001703007
95	0.999818	0.999909	0.000034	0.0000337102	0.0000000011	0.319284255	0.001693875
96	0.999817	0.999908	0.000034	0.0000000016	0.0000000000	0.320960299	0.001684889
97	0.999815	0.999907	0.000034	0.0000337102	0.0000000011	0.322627636	0.001676044
98	0.999813	0.999906	0.000034	0.0000000016	0.0000000000	0.324286401	0.001667337
99	0.999811	0.999905	0.000034	0.0000337102	0.0000000011	0.325936725	0.001658765
100	0.999809	0.999904	0.000034	0.0000000016	0.0000000000	0.327578733	0.001650323

$r_0 = 10 \text{ cm}$

N	$\cos \theta_n$	$q(\theta_n)$	E_{sn}	R_{sn}	I_n	ρ_n	w_n
1	1.000000	1.000000	0.000135	0.0001348470	0.0000000182	0.020717898
2	0.999995	0.999998	0.000135	0.0000000003	0.0000000000	0.029299533	0.020717898
3	0.999990	0.999995	0.000135	0.0001348467	0.0000000182	0.035884452	0.008581634
4	0.999986	0.999993	0.000135	0.0000000007	0.0000000000	0.041435796	0.006584920
5	0.999981	0.999990	0.000135	0.0001348463	0.0000000182	0.046326629	0.005551344
6	0.999976	0.999988	0.000135	0.0000000010	0.0000000000	0.050748279	0.004890832
7	0.999971	0.999986	0.000135	0.0001348460	0.0000000182	0.054814406	0.004421650
8	0.999966	0.999983	0.000135	0.0000000013	0.0000000000	0.058599065	0.004066127
9	0.999961	0.999981	0.000135	0.0001348457	0.0000000182	0.062153695	0.003784659
10	0.999957	0.999978	0.000135	0.0000000016	0.0000000000	0.065515747	0.003554629
11	0.999952	0.999976	0.000135	0.0001348454	0.0000000182	0.068713495	0.003362052
12	0.999947	0.999973	0.000135	0.0000000020	0.0000000000	0.071768905	0.003197748
13	0.999942	0.999971	0.000135	0.0001348450	0.0000000182	0.074699444	0.003055410
14	0.999937	0.999969	0.000135	0.0000000023	0.0000000000	0.077519277	0.002930540
15	0.999932	0.999966	0.000135	0.0001348447	0.0000000182	0.080240075	0.002819833
16	0.999928	0.999964	0.000135	0.0000000026	0.0000000000	0.082871593	0.002720798
17	0.999923	0.999961	0.000135	0.0001348444	0.0000000182	0.085422083	0.002631518
18	0.999918	0.999959	0.000135	0.0000000029	0.0000000000	0.087898598	0.002550490
19	0.999913	0.999957	0.000135	0.0001348441	0.0000000182	0.090307225	0.002476515
20	0.999908	0.999954	0.000135	0.0000000033	0.0000000000	0.092653258	0.002408627
21	0.999903	0.999952	0.000135	0.0001348437	0.0000000182	0.094941337	0.002346033
22	0.999899	0.999949	0.000135	0.0000000036	0.0000000000	0.097175556	0.002288079
23	0.999894	0.999947	0.000135	0.0001348434	0.0000000182	0.099359549	0.002234219
24	0.999889	0.999944	0.000135	0.0000000039	0.0000000000	0.101496558	0.002183993
25	0.999884	0.999942	0.000135	0.0001348431	0.0000000182	0.103589491	0.002137009
26	0.999879	0.999940	0.000135	0.0000000042	0.0000000000	0.105640967	0.002092933
27	0.999874	0.999937	0.000135	0.0001348428	0.0000000182	0.107653357	0.002051476
28	0.999870	0.999935	0.000135	0.0000000046	0.0000000000	0.109628813	0.002012390
29	0.999865	0.999932	0.000135	0.0001348424	0.0000000182	0.111569296	0.001975456
30	0.999860	0.999930	0.000135	0.0000000049	0.0000000000	0.113476602	0.001940484
31	0.999855	0.999928	0.000135	0.0001348421	0.0000000182	0.115352375	0.001907306
32	0.999850	0.999925	0.000135	0.0000000052	0.0000000000	0.117198131	0.001875773
33	0.999845	0.999923	0.000135	0.0001348418	0.0000000182	0.119015264	0.001845755
34	0.999841	0.999920	0.000135	0.0000000055	0.0000000000	0.120805068	0.001817134
35	0.999836	0.999918	0.000135	0.0001348415	0.0000000182	0.122568739	0.001789804
36	0.999831	0.999916	0.000135	0.0000000059	0.0000000000	0.124307389	0.001763671
37	0.999826	0.999913	0.000135	0.0001348411	0.0000000182	0.126022055	0.001738651
38	0.999821	0.999911	0.000135	0.0000000062	0.0000000000	0.127713702	0.001714666
39	0.999817	0.999908	0.000135	0.0001348408	0.0000000182	0.129383233	0.001691647
40	0.999812	0.999906	0.000135	0.0000000065	0.0000000000	0.131031493	0.001669531
41	0.999807	0.999903	0.000135	0.0001348405	0.0000000182	0.132659276	0.001648260
42	0.999802	0.999901	0.000135	0.0000000068	0.0000000000	0.134267326	0.001627783
43	0.999797	0.999899	0.000135	0.0001348402	0.0000000182	0.135856344	0.001608050
44	0.999792	0.999896	0.000135	0.0000000072	0.0000000000	0.137426990	0.001589018
45	0.999788	0.999894	0.000135	0.0001348398	0.0000000182	0.138979886	0.001570646
46	0.999783	0.999891	0.000135	0.0000000075	0.0000000000	0.140515622	0.001552897
47	0.999778	0.999889	0.000135	0.0001348395	0.0000000182	0.142034754	0.001535736
48	0.999773	0.999887	0.000135	0.0000000078	0.0000000000	0.143537809	0.001519132
49	0.999768	0.999884	0.000135	0.0001348392	0.0000000182	0.145025288	0.001503055
50	0.999763	0.999882	0.000135	0.0000000081	0.0000000000	0.146497663	0.001487478

$r_0 = 10 \text{ cm}$

N	$\cos \theta_n$	$q(\theta_n)$	E_{sn}	R_{sn}	I_n	ρ_n	w_n
51	0.999759	0.099879	0.000135	0.0001348389	0.0000000182	0.147955387	0.001472376
52	0.999754	0.999877	0.000135	0.0000000085	0.0000000000	0.149398889	0.001457724
53	0.999749	0.999874	0.000135	0.0001348385	0.0000000182	0.150828576	0.001443501
54	0.999744	0.999872	0.000135	0.0000000088	0.0000000000	0.152244838	0.001429687
55	0.999739	0.999870	0.000135	0.0001348382	0.0000000182	0.153648045	0.001416262
56	0.999734	0.999867	0.000135	0.0000000091	0.0000000000	0.155038554	0.001403208
57	0.999730	0.999865	0.000135	0.0001348379	0.0000000182	0.156416701	0.001390508
58	0.999725	0.999862	0.000135	0.0000000094	0.0000000000	0.157782812	0.001378148
59	0.999720	0.999860	0.000135	0.0001348376	0.0000000182	0.159137196	0.001366111
60	0.999715	0.999858	0.000135	0.0000000098	0.0000000000	0.160480150	0.001354384
61	0.999710	0.999855	0.000135	0.0001348372	0.0000000182	0.161811958	0.001342954
62	0.999705	0.999853	0.000135	0.0000000101	0.0000000000	0.163132894	0.001331808
63	0.999701	0.999850	0.000135	0.0001348369	0.0000000182	0.164443219	0.001320936
64	0.999696	0.999848	0.000135	0.0000000104	0.0000000000	0.165743186	0.001310325
65	0.999691	0.999846	0.000135	0.0001348366	0.0000000182	0.167033035	0.001299967
66	0.999686	0.999843	0.000135	0.0000000107	0.0000000000	0.168313001	0.001289850
67	0.999681	0.999841	0.000135	0.0001348362	0.0000000182	0.169583306	0.001279965
68	0.999677	0.999838	0.000135	0.0000000111	0.0000000000	0.170844165	0.001270305
69	0.999672	0.999836	0.000135	0.0001348359	0.0000000182	0.172095788	0.001260860
70	0.999667	0.999833	0.000135	0.0000000114	0.0000000000	0.173338373	0.001251622
71	0.999662	0.999831	0.000135	0.0001348356	0.0000000182	0.174572113	0.001242585
72	0.999657	0.999829	0.000135	0.0000000117	0.0000000000	0.175797196	0.001233741
73	0.999652	0.999826	0.000135	0.0001348353	0.0000000182	0.177013800	0.001225082
74	0.999648	0.999824	0.000135	0.0000000120	0.0000000000	0.178222099	0.001216604
75	0.999643	0.999821	0.000135	0.0001348349	0.0000000182	0.179422262	0.001208299
76	0.999638	0.999819	0.000135	0.0000000124	0.0000000000	0.180614449	0.001200162
77	0.999633	0.999817	0.000135	0.0001348346	0.0000000182	0.181798819	0.001192188
78	0.999628	0.999814	0.000135	0.0000000127	0.0000000000	0.182975523	0.001184370
79	0.999623	0.999812	0.000135	0.0001348343	0.0000000182	0.184144707	0.001176704
80	0.999619	0.999809	0.000135	0.0000000130	0.0000000000	0.185306515	0.001169185
81	0.999614	0.999807	0.000135	0.0001348340	0.0000000182	0.186461084	0.001161808
82	0.999609	0.999804	0.000135	0.0000000133	0.0000000000	0.187608548	0.001154569
83	0.999604	0.999802	0.000135	0.0001348336	0.0000000182	0.188749036	0.001147464
84	0.999599	0.999800	0.000135	0.0000000137	0.0000000000	0.189882674	0.001140488
85	0.999594	0.999797	0.000135	0.0001348333	0.0000000182	0.191009584	0.001133638
86	0.999590	0.999795	0.000135	0.0000000140	0.0000000000	0.192129884	0.001126910
87	0.999585	0.999792	0.000135	0.0001348330	0.0000000182	0.193243690	0.001120300
88	0.999580	0.999790	0.000135	0.0000000143	0.0000000000	0.194351113	0.001113806
89	0.999575	0.999788	0.000135	0.0001348327	0.0000000182	0.195452261	0.001107423
90	0.999570	0.999785	0.000135	0.0000000146	0.0000000000	0.196547240	0.001101148
91	0.999566	0.999783	0.000135	0.0001348323	0.0000000182	0.197636153	0.001094979
92	0.999561	0.999780	0.000135	0.0000000150	0.0000000000	0.198719099	0.001088913
93	0.999556	0.999778	0.000135	0.0001348320	0.0000000182	0.199796175	0.001082946
94	0.999551	0.999776	0.000135	0.0000000153	0.0000000000	0.200867476	0.001077076
95	0.999546	0.999773	0.000135	0.0001348317	0.0000000182	0.201933093	0.001071301
96	0.999541	0.999771	0.000135	0.0000000156	0.0000000000	0.202993117	0.001065617
97	0.999537	0.999768	0.000135	0.0001348314	0.0000000182	0.204047634	0.001060024
98	0.999532	0.999766	0.000135	0.0000000159	0.0000000000	0.205096729	0.001054517
99	0.999527	0.999763	0.000135	0.0001348310	0.0000000182	0.206140485	0.001049095
100	0.999522	0.999761	0.000135	0.0000000163	0.0000000000	0.207178982	0.001043756

$r_0 = 1 \text{ cm}$

N	$\cos \theta_n$	$q(\theta_n)$	E_{sn}	R_{sn}	I_n	ρ_n	w_n
1	1.000000	1.000000	0.000193	0.0001926386	0.0000000371	0.007830629
2	0.999966	0.999983	0.000193	0.0000000033	0.0000000000	0.011074182	0.007830629
3	0.999932	0.999966	0.000193	0.0001926353	0.0000000371	0.013563048	0.003243553
4	0.999899	0.999949	0.000193	0.0000000065	0.0000000000	0.015661259	0.002488866
5	0.999865	0.999932	0.000193	0.0001926320	0.0000000371	0.017509820	0.002098211
6	0.999831	0.999916	0.000193	0.0000000098	0.0000000000	0.019181047	0.001848561
7	0.999797	0.999899	0.000193	0.0001926288	0.0000000371	0.020717898	0.001671227
8	0.999763	0.999882	0.000193	0.0000000130	0.0000000000	0.022148365	0.001536852
9	0.999730	0.999865	0.000193	0.0001926255	0.0000000371	0.023491888	0.001430467
10	0.999696	0.999848	0.000193	0.0000000163	0.0000000000	0.024762625	0.001343524
11	0.999662	0.999831	0.000193	0.0001926223	0.0000000371	0.025971260	0.001270736
12	0.999628	0.999814	0.000193	0.0000000195	0.0000000000	0.027126096	0.001208635
13	0.999595	0.999797	0.000193	0.0001926190	0.0000000371	0.028233736	0.001154836
14	0.999561	0.999780	0.000193	0.0000000228	0.0000000000	0.029299533	0.001107640
15	0.999527	0.999763	0.000193	0.0001926158	0.0000000371	0.030327898	0.001065797
16	0.999493	0.999747	0.000193	0.0000000260	0.0000000000	0.031322518	0.001028365
17	0.999459	0.999730	0.000193	0.0001926125	0.0000000371	0.032286512	0.000994620
18	0.999426	0.999713	0.000193	0.0000000293	0.0000000000	0.033222547	0.000963995
19	0.999392	0.999696	0.000193	0.0001926093	0.0000000371	0.034132923	0.000936035
20	0.999358	0.999679	0.000193	0.0000000325	0.0000000000	0.035019640	0.000910375
21	0.999324	0.999662	0.000193	0.0001926060	0.0000000371	0.035884452	0.000886717
22	0.999291	0.999645	0.000193	0.0000000358	0.0000000000	0.036728908	0.000864813
23	0.999257	0.999628	0.000193	0.0001926028	0.0000000371	0.037554380	0.000844456
24	0.999223	0.999612	0.000193	0.0000000390	0.0000000000	0.038362093	0.000825472
25	0.999189	0.999595	0.000193	0.0001925995	0.0000000371	0.039153147	0.000807713
26	0.999156	0.999578	0.000193	0.0000000423	0.0000000000	0.039928533	0.000791054
27	0.999122	0.999561	0.000193	0.0001925963	0.0000000371	0.040689144	0.000775385
28	0.999088	0.999544	0.000193	0.0000000455	0.0000000000	0.041435796	0.000760612
29	0.999054	0.999527	0.000193	0.0001925930	0.0000000371	0.042169230	0.000746652
30	0.999021	0.999510	0.000193	0.0000000488	0.0000000000	0.042890124	0.000733434
31	0.998987	0.999493	0.000193	0.0001925898	0.0000000371	0.043599100	0.000720894
32	0.998953	0.999477	0.000193	0.0000000520	0.0000000000	0.044296730	0.000708976
33	0.998919	0.999460	0.000193	0.0001925865	0.0000000371	0.044983542	0.000697630
34	0.998886	0.999443	0.000193	0.0000000553	0.0000000000	0.045660024	0.000686812
35	0.998852	0.999426	0.000193	0.0001925833	0.0000000371	0.046326629	0.000676482
36	0.998818	0.999409	0.000193	0.0000000585	0.0000000000	0.046983777	0.000666605
37	0.998785	0.999392	0.000193	0.0001925800	0.0000000371	0.047631860	0.000657148
38	0.998751	0.999375	0.000193	0.0000000618	0.0000000000	0.048271242	0.000648083
39	0.998717	0.999359	0.000193	0.0001925768	0.0000000371	0.048902265	0.000639382
40	0.998683	0.999342	0.000193	0.0000000650	0.0000000000	0.049525249	0.000631023
41	0.998650	0.999325	0.000193	0.0001925735	0.0000000371	0.050140493	0.000622984
42	0.998616	0.999308	0.000193	0.0000000683	0.0000000000	0.050748279	0.000615244
43	0.998582	0.999291	0.000193	0.0001925703	0.0000000371	0.051348871	0.000607786
44	0.998549	0.999274	0.000193	0.0000000715	0.0000000000	0.051942520	0.000600592
45	0.998515	0.999257	0.000193	0.0001925670	0.0000000371	0.052529459	0.000593648
46	0.998481	0.999241	0.000193	0.0000000748	0.0000000000	0.053109913	0.000586940
47	0.998447	0.999224	0.000193	0.0001925638	0.0000000371	0.053684091	0.000580454
48	0.998414	0.999207	0.000193	0.0000000780	0.0000000000	0.054252192	0.000574178
49	0.998380	0.999190	0.000193	0.0001925605	0.0000000371	0.054814406	0.000568101
50	0.998346	0.999173	0.000193	0.0000000813	0.0000000000	0.055370912	0.000562214

$r_0 = 1 \text{ cm}$

N	$\cos \theta_n$	$q(\theta_n)$	E_{sn}	R_{so}	I_n	ρ_n	w_n
51	0.998313	0.999156	0.000192	0.0001925573	0.0000000371	0.055921880	0.000556506
52	0.998279	0.999139	0.000192	0.0000000845	0.0000000000	0.056467472	0.000550968
53	0.998245	0.999123	0.000192	0.0001925541	0.0000000371	0.057007843	0.000545592
54	0.998212	0.999106	0.000192	0.0000000878	0.0000000000	0.057543140	0.000540371
55	0.998178	0.999089	0.000192	0.0001925508	0.0000000371	0.058073503	0.000535297
56	0.998144	0.999072	0.000192	0.0000000910	0.0000000000	0.058599065	0.000530363
57	0.998110	0.999055	0.000192	0.0001925476	0.0000000371	0.059119956	0.000525563
58	0.998077	0.999038	0.000192	0.0000000942	0.0000000000	0.056366297	0.000520891
59	0.998043	0.999022	0.000192	0.0001925443	0.0000000371	0.060148206	0.000516341
60	0.998009	0.999005	0.000192	0.0000000975	0.0000000000	0.060655795	0.000511909
61	0.997976	0.998988	0.000192	0.0001925411	0.0000000371	0.061159171	0.000507589
62	0.997942	0.998971	0.000192	0.0000001007	0.0000000000	0.061658438	0.000503376
63	0.997908	0.998954	0.000192	0.0001925378	0.0000000371	0.062153695	0.000499267
64	0.997875	0.998937	0.000192	0.0000001040	0.0000000000	0.062645036	0.000495256
65	0.997841	0.998921	0.000192	0.0001925346	0.0000000371	0.063132553	0.000491341
66	0.997807	0.998904	0.000192	0.0000001072	0.0000000000	0.063616335	0.000487517
67	0.997774	0.998887	0.000192	0.0001925314	0.0000000371	0.064096465	0.000483781
68	0.997740	0.998870	0.000192	0.0000001105	0.0000000000	0.064573025	0.000480130
69	0.997706	0.998853	0.000192	0.0001925281	0.0000000371	0.065046094	0.000476560
70	0.997673	0.998836	0.000192	0.0000001137	0.0000000000	0.065515747	0.000473069
71	0.997639	0.998820	0.000192	0.0001925249	0.0000000371	0.065982057	0.000469653
72	0.997606	0.998803	0.000192	0.0000001169	0.0000000000	0.066445094	0.000466310
73	0.997572	0.998786	0.000192	0.0001925216	0.0000000371	0.066904928	0.000463038
74	0.997538	0.998769	0.000192	0.0000001202	0.0000000000	0.067361622	0.000459833
75	0.997505	0.998752	0.000192	0.0001925184	0.0000000371	0.067815241	0.000456694
76	0.997471	0.998735	0.000192	0.0000001234	0.0000000000	0.068265845	0.000453619
77	0.997437	0.998719	0.000192	0.0001925151	0.0000000371	0.068713493	0.000450605
78	0.997404	0.998702	0.000192	0.0000001267	0.0000000000	0.069158247	0.000447650
79	0.997370	0.998685	0.000192	0.0001925119	0.0000000371	0.069600157	0.000444752
80	0.997336	0.998668	0.000192	0.0000001299	0.0000000000	0.070039279	0.000441910
81	0.997303	0.998651	0.000192	0.0001925087	0.0000000371	0.070475665	0.000439122
82	0.997269	0.998635	0.000192	0.0000001331	0.0000000000	0.070909366	0.000436386
83	0.997236	0.998618	0.000192	0.0001925054	0.0000000371	0.071340430	0.000433700
84	0.997202	0.998601	0.000192	0.0000001364	0.0000000000	0.071768905	0.000431064
85	0.997168	0.998584	0.000192	0.0001925022	0.0000000371	0.072194837	0.000428475
86	0.997135	0.998567	0.000192	0.0000001396	0.0000000000	0.072618270	0.000425932
87	0.997101	0.998551	0.000192	0.0001924990	0.0000000371	0.073039249	0.000423434
88	0.997067	0.998534	0.000192	0.0000001429	0.0000000000	0.073457816	0.000420979
89	0.997034	0.998517	0.000192	0.0001924957	0.0000000371	0.073874011	0.000418566
90	0.997000	0.998500	0.000192	0.0000001461	0.0000000000	0.074287874	0.000416195
91	0.996967	0.998483	0.000192	0.0001924925	0.0000000371	0.074699444	0.000413863
92	0.996933	0.998467	0.000192	0.0000001493	0.0000000000	0.075108759	0.000411570
93	0.996899	0.998450	0.000192	0.0001924892	0.0000000371	0.075515856	0.000409315
94	0.996866	0.998433	0.000192	0.0000001526	0.0000000000	0.075920770	0.000407097
95	0.996832	0.998416	0.000192	0.0001924860	0.0000000371	0.076323535	0.000404914
96	0.996799	0.998399	0.000192	0.0000001558	0.0000000000	0.076724186	0.000402766
97	0.996765	0.998383	0.000192	0.0001924828	0.0000000371	0.077122756	0.000400651
98	0.996731	0.998366	0.000192	0.0000001590	0.0000000000	0.077519277	0.000398570
99	0.996698	0.998349	0.000192	0.0001924795	0.0000000371	0.077913780	0.000396521
100	0.996664	0.998332	0.000192	0.0000001623	0.0000000000	0.078306295	0.000394503

$r_0 = 0.01 \text{ cm}$

N	$\cos \theta_n$	$q(\theta_n)$	E_{sn}	R_{sn}	I_n	ρ_n	w_n
1	1.000000	1.000000	0.000202	0.0002021694	0.0000000409	0.000802200
2	0.996789	0.998395	0.000202	0.0000003245	0.0000000000	0.001134482	0.000802200
3	0.993600	0.996800	0.000202	0.0002018469	0.0000000407	0.001389452	0.000332282
4	0.990430	0.995215	0.000201	0.0000006449	0.0000000000	0.001604400	0.000254969
5	0.987280	0.993640	0.000201	0.0002015286	0.0000000406	0.001793774	0.000214949
6	0.984151	0.992075	0.000201	0.0000009613	0.0000000000	0.001964981	0.000189374
7	0.981041	0.990521	0.000200	0.0002012142	0.0000000405	0.002122422	0.000171207
8	0.977951	0.988975	0.000200	0.0000012737	0.0000000000	0.002268963	0.000157441
9	0.974880	0.987440	0.000200	0.0002009038	0.0000000404	0.002406601	0.000146543
10	0.971829	0.985914	0.000199	0.0000015821	0.0000000000	0.002536780	0.000137636
11	0.968796	0.984398	0.000199	0.0002005973	0.0000000402	0.002660597	0.000130179
12	0.965782	0.982891	0.000199	0.0000018868	0.0000000000	0.002778903	0.000123817
13	0.962787	0.981394	0.000198	0.0002002945	0.0000000401	0.002892374	0.000118306
14	0.959811	0.979905	0.000198	0.0000021876	0.0000000000	0.003001558	0.000113471
15	0.956853	0.978426	0.000198	0.0001999955	0.0000000400	0.003106908	0.000109184
16	0.953913	0.976956	0.000198	0.0000024848	0.0000000000	0.003208801	0.000105350
17	0.950991	0.975495	0.000197	0.0001997001	0.0000000399	0.003307556	0.000101893
18	0.948087	0.974043	0.000197	0.0000027784	0.0000000000	0.003403447	0.000098755
19	0.945200	0.972600	0.000197	0.0001994084	0.0000000398	0.002496710	0.000095891
20	0.942331	0.971166	0.000196	0.0000030684	0.0000000000	0.003587548	0.000093262
21	0.939480	0.969740	0.000196	0.0001991201	0.0000000396	0.003676143	0.000090839
22	0.936645	0.968323	0.000196	0.0000033549	0.0000000000	0.003762653	0.000088595
23	0.933828	0.966914	0.000195	0.0001988353	0.0000000395	0.003847217	0.000086509
24	0.931027	0.965514	0.000195	0.0000036380	0.0000000000	0.003929962	0.000084565
25	0.928244	0.964122	0.000195	0.0001985539	0.0000000394	0.004011001	0.000082745
26	0.925477	0.962738	0.000195	0.0000039177	0.0000000000	0.004090435	0.000081039
27	0.922726	0.961363	0.000194	0.0001982759	0.0000000393	0.004168355	0.000079433
28	0.919991	0.959996	0.000194	0.0000041941	0.0000000000	0.004244845	0.000077920
29	0.917273	0.958637	0.000194	0.0001980011	0.0000000392	0.004319980	0.000076490
30	0.914571	0.957285	0.000194	0.0000044673	0.0000000000	0.004393832	0.000075136
31	0.911885	0.955942	0.000193	0.0001977295	0.0000000391	0.004466462	0.000073851
32	0.909214	0.954607	0.000193	0.0000047372	0.0000000000	0.004537930	0.000072630
33	0.906559	0.953279	0.000193	0.0001974612	0.0000000390	0.004608289	0.000071468
34	0.903919	0.951960	0.000192	0.0000050041	0.0000000000	0.004677591	0.000070360
35	0.901295	0.950647	0.000192	0.0001971959	0.0000000389	0.004745881	0.000069301
36	0.898686	0.949343	0.000192	0.0000057678	0.0000000000	0.004813201	0.000068290
37	0.896092	0.948046	0.000192	0.0001969337	0.0000000388	0.004879593	0.000067321
38	0.893513	0.946756	0.000191	0.0000055285	0.0000000000	0.004945094	0.000066392
39	0.890948	0.945474	0.000191	0.0001966745	0.0000000387	0.005009739	0.000065501
40	0.888399	0.944199	0.000191	0.0000057862	0.0000000000	0.005073560	0.000064644
41	0.885864	0.942932	0.000191	0.0001964182	0.0000000386	0.005136588	0.000063821
42	0.883343	0.941671	0.000190	0.0000060410	0.0000000000	0.005198852	0.000063028
43	0.880836	0.940418	0.000190	0.0001961648	0.0000000385	0.005260379	0.000062264
44	0.878344	0.939172	0.000190	0.0000062930	0.0000000000	0.005321194	0.000061527
45	0.875866	0.937933	0.000190	0.0001959143	0.0000000384	0.005381323	0.000060816
46	0.873402	0.936701	0.000189	0.0000065421	0.0000000000	0.005440787	0.000060128
47	0.870951	0.935476	0.000189	0.0001956666	0.0000000383	0.005499608	0.000059464
48	0.868515	0.934257	0.000189	0.0000067884	0.0000000000	0.0055578060	0.000058821
49	0.866091	0.933046	0.000189	0.0001954217	0.0000000382	0.002615402	0.000058199
50	0.863682	0.931841	0.000188	0.0000070320	0.0000000000	0.002672412	0.000057595

$$r_0 = 0.01 \text{ cm}$$

N	$\cos \theta_n$	$q(\theta_n)$	E_{sn}	R_{sn}	I_n	ρ_n	w_n
51	0.861286	0.930643	0.000188	0.0001951795	0.0000000381	0.005728855	0.000057011
52	0.858902	0.929451	0.000188	0.0000072729	0.0000000001	0.005784748	0.000056443
53	0.856533	0.928266	0.000188	0.0001949399	0.0000000380	0.005840106	0.000055893
54	0.854176	0.927088	0.000187	0.0000075111	0.0000000001	0.005894944	0.000055358
55	0.851832	0.925916	0.000187	0.0001947030	0.0000000379	0.005949276	0.000054838
56	0.849501	0.924750	0.000187	0.0000077467	0.0000000001	0.006003117	0.000054332
57	0.847182	0.923591	0.000187	0.0001944686	0.0000000378	0.006056479	0.000053841
58	0.844876	0.922438	0.000186	0.0000079798	0.0000000001	0.006109375	0.000053362
59	0.842583	0.921292	0.000186	0.0001942368	0.0000000377	0.006161817	0.000052896
60	0.840302	0.920151	0.000186	0.0000082104	0.0000000001	0.006213816	0.000052442
61	0.838034	0.919017	0.000186	0.0001940075	0.0000000376	0.006265384	0.000051999
62	0.835777	0.917889	0.000186	0.0000084385	0.0000000001	0.006316531	0.000051568
63	0.833533	0.916767	0.000185	0.0001937806	0.0000000376	0.006367267	0.000051147
64	0.831301	0.915650	0.000185	0.0000086641	0.0000000001	0.006417602	0.000050736
65	0.829081	0.914540	0.000185	0.0001935562	0.0000000375	0.006467545	0.000050335
66	0.826872	0.913436	0.000185	0.0000088873	0.0000000001	0.006517105	0.000049943
67	0.824675	0.912338	0.000184	0.0001933341	0.0000000374	0.006566292	0.000049560
68	0.822490	0.911245	0.000184	0.0000091082	0.0000000001	0.006615113	0.000049186
69	0.820317	0.910158	0.000184	0.0001931144	0.0000000373	0.006663576	0.000048821
70	0.818155	0.909077	0.000184	0.0000093268	0.0000000001	0.006711689	0.000048463
71	0.816004	0.908002	0.000184	0.0001928970	0.0000000372	0.006759459	0.000048113
72	0.813864	0.906932	0.000183	0.0000095431	0.0000000001	0.006806895	0.000047771
73	0.811736	0.905868	0.000183	0.0001926818	0.0000000371	0.006854002	0.000047435
74	0.809619	0.904809	0.000183	0.0000097571	0.0000000001	0.006900787	0.000047107
75	0.807513	0.903756	0.000183	0.0001924689	0.0000000370	0.006947258	0.000046786
76	0.805417	0.902709	0.000183	0.0000099689	0.0000000001	0.006993419	0.000046470
77	0.803333	0.901666	0.000182	0.0001922582	0.0000000370	0.007039278	0.000046162
78	0.901259	0.900630	0.000182	0.0000101785	0.0000000001	0.007084841	0.000045859
79	0.799196	0.899598	0.000182	0.0001920497	0.0000000369	0.007130112	0.000045562
80	0.797144	0.898572	0.000182	0.0000103860	0.0000000001	0.007175097	0.000045271
81	0.795102	0.897551	0.000181	0.0001918433	0.0000000368	0.007219802	0.000044985
82	0.793070	0.896535	0.000181	0.0000105913	0.0000000001	0.007264232	0.000044705
83	0.791049	0.895525	0.000181	0.0001916390	0.0000000367	0.007308392	0.000044430
84	0.789038	0.894519	0.000181	0.0000107946	0.0000000001	0.007322860	0.000044160
85	0.787037	0.893519	0.000181	0.0001914367	0.0000000366	0.007395921	0.000043895
86	0.785047	0.892523	0.000180	0.0000109958	0.0000000001	0.007439299	0.000043634
87	0.783066	0.891533	0.000180	0.0001912365	0.0000000366	0.007482426	0.000043378
88	0.781096	0.890548	0.000180	0.0000111950	0.0000000001	0.007525305	0.000043127
89	0.779135	0.889568	0.000180	0.0001910383	0.0000000365	0.007367942	0.000042880
90	0.777184	0.888592	0.000180	0.0000113922	0.0000000001	0.007610340	0.000042637
91	0.775243	0.887622	0.000179	0.0001908421	0.0000000364	0.007652502	0.000042398
92	0.773312	0.886656	0.000179	0.0000115874	0.0000000001	0.007694434	0.000042163
93	0.771390	0.885695	0.000179	0.0001906478	0.0000000363	0.007736139	0.000041932
94	0.769477	0.884739	0.000179	0.0000117808	0.0000000001	0.007777620	0.000041705
95	0.767574	0.883787	0.000179	0.0001904555	0.0000000363	0.007818881	0.000041481
96	0.765681	0.882840	0.000178	0.0000119722	0.0000000001	0.007859925	0.000041261
97	0.763797	0.881898	0.000178	0.0001902650	0.0000000362	0.007900756	0.000041044
98	0.761922	0.880961	0.000178	0.0000121617	0.0000000001	0.007941377	0.000040831
99	0.760056	0.880028	0.000178	0.0001900764	0.0000000361	0.007981791	0.000040621
100	0.758199	0.879100	0.000178	0.0000123494	0.0000000002	0.008022002	0.000040414

$r_0 = 0.0001 \text{ cm}$

N	$\cos \theta_n$	$q(\theta_n)$	E_{sn}	R_{sn}	I_n	ρ_n	w_n
1	1.000000	1.000000	0.000202	0.0002022695	0.0000000409	0.000080240
2	0.756472	0.878236	0.000178	0.0000246291	0.0000000006	0.000113476	0.000080240
3	0.608327	0.804164	0.000163	0.0001872869	0.0000000351	0.000138980	0.000033236
4	0.508704	0.754352	0.000153	0.0000347045	0.0000000012	0.000160480	0.000025503
5	0.437119	0.718559	0.000145	0.0001800471	0.0000000324	0.000179422	0.000021500
6	0.383195	0.691597	0.000140	0.0000401581	0.0000000016	0.000196547	0.000018942
7	0.341114	0.670557	0.000136	0.0001757913	0.0000000309	0.000212295	0.000017125
8	0.307361	0.653681	0.000132	0.0000435717	0.0000000019	0.000226953	0.000015748
9	0.279686	0.639843	0.000129	0.0001729924	0.0000000299	0.000240720	0.000014658
10	0.256583	0.628292	0.000127	0.0000459082	0.0000000021	0.000253741	0.000013767
11	0.237006	0.618503	0.000125	0.0001710124	0.0000000292	0.000266126	0.000013021
12	0.220204	0.610102	0.000123	0.0000476074	0.0000000023	0.000277959	0.000012385
13	0.205627	0.602813	0.000122	0.0001695382	0.0000000287	0.000289309	0.000011834
14	0.192860	0.596430	0.000121	0.0000488986	0.0000000024	0.000300230	0.000011350
15	0.181585	0.590793	0.000119	0.0001683979	0.0000000284	0.000310768	0.000010921
16	0.171556	0.585778	0.000118	0.0000499129	0.0000000025	0.000320959	0.000010538
17	0.162577	0.581288	0.000118	0.0001674898	0.0000000281	0.000330837	0.000010192
18	0.154491	0.577245	0.000117	0.0000507307	0.0000000026	0.000340429	0.000009878
19	0.147171	0.573585	0.000116	0.0001667495	0.0000000278	0.000349758	0.000009591
20	0.140513	0.570257	0.000115	0.0000514040	0.0000000026	0.000358844	0.000009329
21	0.134432	0.567216	0.000115	0.0001661345	0.0000000276	0.000367705	0.000009086
22	0.128855	0.564428	0.000114	0.0000519680	0.0000000027	0.000376358	0.000008862
23	0.123723	0.561861	0.000114	0.0001656154	0.0000000274	0.000384817	0.000008653
24	0.118983	0.559492	0.000113	0.0000524473	0.0000000028	0.000393093	0.000008459
25	0.114593	0.557297	0.000113	0.0001651714	0.0000000273	0.000401199	0.000008277
26	0.110516	0.555258	0.000112	0.0000528597	0.0000000028	0.000409143	0.000008106
27	0.106719	0.553360	0.000112	0.0001647874	0.0000000272	0.000416939	0.000007945
28	0.103174	0.551587	0.000112	0.0000532182	0.0000000028	0.000424590	0.000007794
29	0.099857	0.549929	0.000111	0.0001644520	0.0000000270	0.000432105	0.000007651
30	0.096747	0.548373	0.000111	0.0000535328	0.0000000029	0.000439492	0.000007515
31	0.093824	0.546912	0.000111	0.0001641564	0.0000000269	0.000446757	0.000007387
32	0.091073	0.545537	0.000110	0.0000538110	0.0000000029	0.000453905	0.000007265
33	0.088479	0.544239	0.000110	0.0001638940	0.0000000269	0.000460943	0.000007149
34	0.086028	0.543014	0.000110	0.0000540588	0.0000000029	0.000467875	0.000007038
35	0.083710	0.541855	0.000110	0.0001636595	0.0000000268	0.000474705	0.000006932
36	0.081513	0.540756	0.000109	0.0000542810	0.0000000029	0.000481439	0.000006831
37	0.079428	0.539714	0.000109	0.0001634487	0.0000000267	0.000488080	0.000006734
38	0.077448	0.538724	0.000109	0.0000544813	0.0000000030	0.000494632	0.000006641
39	0.075564	0.537782	0.000109	0.0001632582	0.0000000267	0.000501098	0.000006552
40	0.073769	0.536884	0.000109	0.0000546628	0.0000000030	0.000507482	0.000006466
41	0.072057	0.536029	0.000108	0.0001630851	0.0000000266	0.000513786	0.000006384
42	0.070424	0.535212	0.000108	0.0000548281	0.0000000030	0.000520014	0.000006304
43	0.068862	0.534431	0.000108	0.0001629272	0.0000000265	0.000526168	0.000006228
44	0.067368	0.533684	0.000108	0.0000549791	0.0000000030	0.000532251	0.000006154
45	0.065938	0.532969	0.000108	0.0001627825	0.0000000265	0.000538265	0.000006083
46	0.064567	0.532284	0.000108	0.0000551178	0.0000000030	0.000544213	0.000006014
47	0.063253	0.531626	0.000108	0.0001626495	0.0000000265	0.000550097	0.000005948
48	0.061990	0.530995	0.000107	0.0000552454	0.0000000031	0.000555918	0.000005884
49	0.060777	0.530388	0.000107	0.0001625268	0.0000000264	0.000561679	0.000005821
50	0.059610	0.529805	0.000107	0.0000553634	0.0000000031	0.000567382	0.000005761

$r_o = 0.0001 \text{ cm}$

N	$\cos \theta_n$	$q(\theta_n)$	E_{sn}	R_{sn}	I_n	ρ_n	w_n
51	0.058488	0.529244	0.000107	0.0001624133	0.0000000264	0.000573027	0.000005702
52	0.057407	0.528703	0.000107	0.0000554727	0.0000000031	0.000578618	0.000005646
53	0.056365	0.528183	0.000107	0.0001623079	0.0000000263	0.000584155	0.000005591
54	0.055360	0.527680	0.000107	0.0000555744	0.0000000031	0.000589640	0.000005537
55	0.054391	0.527195	0.000107	0.0001622099	0.0000000263	0.000595075	0.000005485
56	0.053455	0.526727	0.000107	0.0000556690	0.0000000031	0.000600460	0.000005435
57	0.052550	0.526275	0.000106	0.0001621184	0.0000000263	0.000605798	0.000005385
58	0.051676	0.525838	0.000106	0.0000557575	0.0000000031	0.000611089	0.000005338
59	0.050830	0.525415	0.000106	0.0001620329	0.0000000263	0.000616334	0.000005291
60	0.050012	0.525006	0.000106	0.0000558402	0.0000000031	0.000621535	0.000005245
61	0.049219	0.524609	0.000106	0.0001619527	0.0000000262	0.000626693	0.000005201
62	0.048451	0.524226	0.000106	0.0000559179	0.0000000031	0.000631809	0.000005158
63	0.047707	0.523853	0.000106	0.0001618775	0.0000000262	0.000636884	0.000005116
64	0.046985	0.523493	0.000106	0.0000559909	0.0000000031	0.000641919	0.000005075
65	0.046285	0.523142	0.000106	0.0001618066	0.0000000262	0.000646915	0.000005035
66	0.045605	0.522803	0.000106	0.0000560596	0.0000000031	0.000651872	0.000004996
67	0.044945	0.522473	0.000106	0.0001617399	0.0000000262	0.000656792	0.000004957
68	0.044304	0.522152	0.000106	0.0000561245	0.0000000031	0.000661675	0.000004920
69	0.043681	0.521840	0.000106	0.0001616769	0.0000000261	0.000666522	0.000004883
70	0.043075	0.521538	0.000106	0.0000561858	0.0000000032	0.000671335	0.000004848
71	0.042486	0.521243	0.000105	0.0001616173	0.0000000261	0.000676113	0.000004812
72	0.041912	0.520956	0.000105	0.0000562437	0.0000000032	0.000680858	0.000004778
73	0.041354	0.520677	0.000105	0.0001615608	0.0000000261	0.000685570	0.000004745
74	0.040811	0.520405	0.000105	0.0000562987	0.0000000032	0.000690249	0.000004712
75	0.040281	0.520141	0.000105	0.0001615073	0.0000000261	0.000694898	0.000004680
76	0.039766	0.519883	0.000105	0.0000563509	0.0000000032	0.000699515	0.000004648
77	0.039263	0.519631	0.000105	0.0001614565	0.0000000261	0.000704102	0.000004617
78	0.038773	0.519386	0.000105	0.0000564004	0.0000000032	0.000708659	0.000004587
79	0.038295	0.519147	0.000105	0.0001614081	0.0000000261	0.000713188	0.000004557
80	0.037828	0.518914	0.000105	0.0000564476	0.0000000032	0.000717687	0.000004528
81	0.037373	0.518686	0.000105	0.0001613621	0.0000000260	0.000722159	0.000004500
82	0.036929	0.518464	0.000105	0.0000564926	0.0000000032	0.000726603	0.000004472
83	0.036495	0.518247	0.000105	0.0001613182	0.0000000260	0.000731020	0.000004444
84	0.036071	0.518035	0.000105	0.0000565354	0.0000000032	0.000735411	0.000004417
85	0.035657	0.517828	0.000105	0.0001612763	0.0000000260	0.000739775	0.000004391
86	0.035252	0.517626	0.000105	0.0000565764	0.0000000032	0.000744114	0.000004364
87	0.034856	0.517428	0.000105	0.0001612363	0.0000000260	0.000748428	0.000004339
88	0.034469	0.517235	0.000105	0.0000566155	0.0000000032	0.000752717	0.000004314
89	0.034091	0.517045	0.000105	0.0001611980	0.0000000260	0.000756981	0.000004289
90	0.033721	0.516860	0.000105	0.0000566529	0.0000000032	0.000761222	0.000004265
91	0.033358	0.516679	0.000105	0.0001611614	0.0000000260	0.000765440	0.000004241
92	0.033004	0.516502	0.000104	0.0000566888	0.0000000032	0.000769634	0.000004217
93	0.032657	0.516328	0.000104	0.0001611263	0.0000000260	0.000773805	0.000004194
94	0.032317	0.516158	0.000104	0.0000567232	0.0000000032	0.000777954	0.000004171
95	0.031984	0.515992	0.000104	0.0001610926	0.0000000260	0.000782082	0.000004149
96	0.031658	0.515829	0.000104	0.0000567561	0.0000000032	0.000786187	0.000004127
97	0.031339	0.515669	0.000104	0.0001610603	0.0000000256	0.000790271	0.000004105
98	0.031025	0.515513	0.000104	0.0000567878	0.0000000032	0.000794334	0.000004084
99	0.030719	0.515359	0.000104	0.0001610292	0.0000000259	0.000798377	0.000004063
100	0.030418	0.515209	0.000104	0.0000568182	0.0000000032	0.000802399	0.000004042

Y 軸 方 向 Fraunhofer 回 折 像

水 野 孝 彦

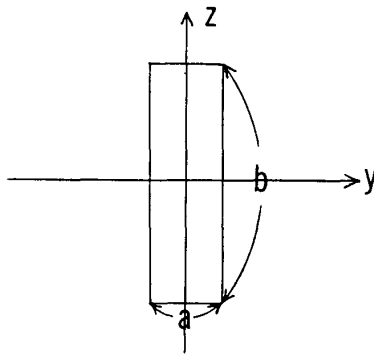
緒 言

下記の如く Fraunhofer 回折像の Y 軸方向を単孔の場合と多重細隙の場合について求め、次に β 方向回折光を求める。

理 論

Y 軸方向回折像

長方形孔の回折像 (単孔)



$$u = Aab \frac{\sin \alpha}{\alpha} \frac{\sin \beta}{\beta}$$

$$I = I_0 \frac{\sin^2 \alpha}{\alpha^2} \frac{\sin^2 \beta}{\beta^2}$$

但し U ; optical disturbance

I ; intensity

$$\alpha \equiv \frac{\pi r_x a}{\lambda} \quad \beta \equiv \frac{\pi r_y b}{\lambda}$$

r_x, r_y ; direction cosins λ ; wave length

Y 軸方向回折像 $\alpha = 0$ とおく.

$$u_y = Aab \frac{\sin \beta}{\beta} \dots \dots a) \quad I_y = I_0 \frac{\sin^2 \beta}{\beta^2} \dots \dots b)$$

暗線の位置 $u = 0 \quad \beta = \pm\pi, \pm 2\pi, \dots, \pm n\pi$

$$Y = \pm \frac{f\lambda}{b}, \pm 2 \frac{f\lambda}{b}, \dots \pm n \frac{f\lambda}{b}$$

明線の位置 u の極大値 $\beta = 0, \pm 1.43\pi, \pm 2.46\pi, \pm 3.47\pi, \pm 4.48\pi$

$$Y = 0, \pm 1.43 \frac{f\lambda}{b}, \pm 2.46 \frac{f\lambda}{b}, \pm 3.47 \frac{f\lambda}{b}, \pm 4.48 \frac{f\lambda}{b}$$

但し Y ; 回折像 (Y 軸) の位置

f ; レンズ (望遠鏡) の焦点距離

明線回折光強度

上式 (b) に明線の位置 β を代入すれば

$$I = I_0, 0.047I_0, 0.017I_0, 0.008I_0$$

(II) 多重スリットに依る Y 軸方向の回折像

$$\begin{aligned} u' &= A \int_{-b/2}^{+b/2} \int_{-\infty}^{+\infty} f(x) \exp[2\pi i(r_x x + r_y y) / \lambda] dy dz \\ &= A \int_{-b/2}^{+b/2} \exp(2\pi i r_y y / \lambda) dz \int_{-\infty}^{+\infty} f(x) \exp(2\pi i r_x x) dx \\ &= Aab \frac{\sin \alpha}{\alpha} \frac{\sin(N\delta)}{\sin \delta} \frac{\sin \beta}{\beta} \end{aligned}$$

但し $\delta = \frac{\pi h \sin \theta}{\lambda}$, h ; スリット間隔

$\delta = h/a \cdot \alpha$ θ ; 回折角

$$I' = \frac{I'_0}{N^2} \frac{\sin^2 \alpha}{\alpha^2} \frac{\sin^2(N\delta)}{\sin^2 \delta} \frac{\sin^2 \beta}{\beta^2}$$

Y 軸方向回折像

前式より $u' = Aab \frac{\sin \alpha}{\alpha} \frac{\sin N\delta}{\sin \delta} \frac{\sin \beta}{\beta} = Aab \frac{\sin \alpha}{\alpha} \frac{\sin N\delta}{N\delta} \frac{N\delta}{\sin \delta} \frac{\sin \beta}{\beta}$

$\alpha = 0$ とおく

$$u'_y = NAab \frac{\sin \beta}{\beta} \dots \dots c) \qquad I'_y = I'_0 \frac{\sin^2 \beta}{\beta^2} \dots \dots d)$$

a) と c) より $u'_y = Nu_y$

両辺を二乗して $u'^2_y = I'_y = N^2 u^2_y$

$I_0 = u^2_y$ なる故

b), d) 式より $I'_0 = N^2 I_0$

明線の位置 $\beta = 0, \beta = \pm 1.43\pi, \pm 2.46\pi, \pm 3.47\pi, \pm 4.48\pi$

$$Z = 0, \pm \frac{1.43f\lambda}{b}, \pm \frac{2.46f\lambda}{b}, \pm \frac{3.47f\lambda}{b}, \pm \frac{4.48f\lambda}{b}, \dots$$

暗線の位置 $\pm \pi, \pm 2\pi \dots \pm n\pi$

$$Z = \pm \frac{f\lambda}{b}, \pm 2\frac{\lambda}{f}, \pm 3\frac{\lambda}{f}, \dots \pm n\frac{\lambda}{f}$$

明線回折光強度 I'_p

上式 d) に β の値を代入すれば

$$I'_p = I'_0, 0.047I'_0, 0.017I'_0, 0.008I'_0$$

(Ⅲ) β 方向回折光総量

$$\begin{aligned} I_{\beta} &= \left[|A|^2 (ab)^2 \int_{-1}^{+1} \left\{ \frac{\sin \alpha}{\alpha} \right\}^2 dr_x \right] \left(\frac{\sin \beta}{\beta} \right)^2 \\ &= (ab |A|^2) (b\lambda) (\sin \beta / \beta)^2 \\ &= I_{\beta 0} (\sin \beta / \beta)^2 \end{aligned}$$

但し I_{β} ; β 方向回折光総量

$$\beta = (\pi/\lambda) b (\sin \theta - \sin \theta_0)$$

$$I_{\beta 0} = ab |A|^2 (b\lambda)$$

従って $I_{\beta} = 0$ の方向は $x = \pm\pi, \pm 2\pi, \pm 3\pi, \pm n\pi$

即ち $\beta = p\lambda/b$

故に暗線の位置 $\sin \theta - \sin \theta_0 = n\lambda/b$ $p = \pm 1, \pm 2, \dots$

明線の位置 $\sin \theta - \sin \theta_0 = n\lambda/b$ $p = 0, \pm 1.43, \pm 2.46, \pm 3.47$

次に I, I_{α}, I_{β} の関係は他日の発表とする.