

On Dynamic Multiple Measuring Methods

—Extension of Gauss' Double Weighing Method for Dynamic Linear Measurement Processes—

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Abstract

The dynamic multiple measuring method, that is, the double measuring method or its improved version, provides not only a precise calibration procedure of linear sensors, but also a linear system identification method whose results are independent of dynamics of measuring devices. The purpose of this tutorial paper is of four folds: 1) to give a brief review of Gauss' double weighing method which has been used for many years as a substitution method for mass measurement; 2) to present a new mathematical interpretations to the double weighing concept, which constitutes the background of its extension to dynamic multiple measuring methods; 3) to describe the influence of measurement noise on the results obtained by the improved double measuring methods; and 4) to suggest their potential applications in linear measurement processes with a few numerical examples.

Keywords: Gauss' double weighing method; linear measuring processes; linear system identification; double measuring method; multiple measuring method; measurement noise; signal-to-noise ratio.

1. Introduction

The differences in dynamics among measuring devices affect the measured results when plural devices are simultaneously used for measurement of physical quantities. The dynamic multiple measuring method, namely; the "double measuring method" and its improved version, the "improved double measuring method", was developed to remove such differences in dynamics from the resulting values [1],[2]. In another word, they provide the methods for relative compensation of the differences in dynamic characteristics of measuring devices. That is; the differences in dynamics among measuring

devices are identified as mathematical models; the identified models are realized as filters; the filters are implemented to the measuring system; and then the differences in dynamics are relatively compensated.

The effectiveness of the relative compensation was proved in an experimental measurement system for calibration of accelerometers [3] and that for a sound transmission loss [4].

These methods are initially proposed under the assumption of a measurement noise being to be ignored. In practice, however, measuring noises cannot be ignored, and analysis of their influence on the resulting values has been

desired.

Accordingly, the influence of measurement noise has also been investigated on the results obtained by applying the improved double measuring methods [5]. The analytical investigation as well as a numerical experiment has then revealed that the influence is appeared as signal-to-noise ratio at an observed output and can be removed in principle when the improved double measuring method is applied to the linear system identification.

The purpose of this tutorial paper is of four folds: 1) to give a brief review of Gauss' double weighing method which has been used for many years as a substitution method for mass measurement; 2) to present a new mathematical interpretations to the double weighing concept, which constitutes the background of its extension to dynamic multiple measuring methods; 3) to describe the influence of measurement noise on the results obtained by the improved double measuring methods; and 4) to suggest their potential applications in linear measurement processes with a few numerical examples.

2. Basic Concept of Gauss' Double Measuring Method

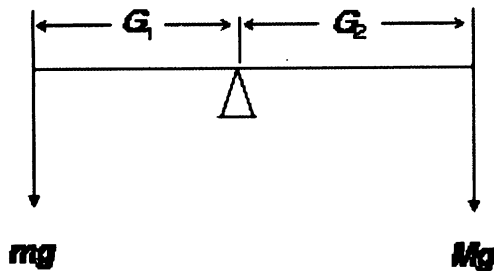


Fig. 1 Schematic of a balance beam in equilibrium

Let us briefly recall of the basic concept of Gauss' double weighing method using the schematic as shown in Fig. 1. It is well known that the fundamental relation for a balance beam in equilibrium is given by

$$mgG_1 = MgG_2, \text{ or } mgG_1 - MgG_2 = 0, \quad \dots (1)$$

where it is assumed that the beam is rigid

body and that the loading points, the fulcrum, and the center of gravity of a balance beam are located on the same line. In Eq. (1), m denotes the mass value of a sample weight to be measured, M is the mass value of standard weight, and g is gravity constant. G_1 and G_2 are the lengths of left and right arms of the balance beam, respectively.

Gauss' double weighing method [1] consists of the following three steps.

1) First measurement:

Putting a sample weight on the left goods-plate of balance, a manipulation is carried out by changing the combination of standard weights on the right goods-plate to settle the balance beam in equilibrium position. In the equilibrium, we get

$$G_1 mg - G_2 M_1 g = 0. \quad \dots (1.a)$$

2) Second measurement:

Putting the sample weight on the right goods-plate, a similar balancing manipulation is conducted by changing the combination of standard weights on the left goods-plate. In the equilibrium, we get

$$G_1 M_2 g - G_2 mg = 0. \quad \dots (1.b)$$

3) Calculation of the mass value:

By means of the balanced mass values, M_1 and M_2 , of standard weights in the first and second measurements, the mass value to be measured is calculated by using the following relation:

$$m = \sqrt{M_1 M_2} \cong \frac{M_1 + M_2}{2}. \quad \dots (2)$$

Remarks 1: As to Eq. (1) we could give a new interpretation that the equation implies the coincidence of output signals from two linear systems whose transfer characteristics are given by G_1 and G_2 , respectively. That is to say, the quantity $G_1 mg$ can be considered as the output value of a linear system G_1 if we consider the mg as the input level to the system. Similarly, the quantity $G_2 Mg$ becomes that of a linear system G_2 for the input level Mg .

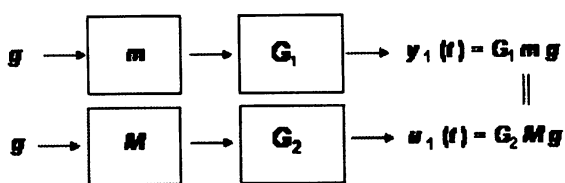


Fig. 2(a) Measurement 1

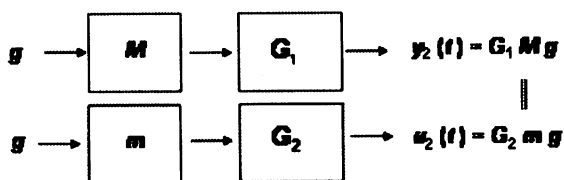


Fig. 2(b) Measurement 2

Figures 2(a) and 2(b) show the block diagram which illustrates the essence of a new interpretation to Gauss' double weighing principle from a system-theoretic viewpoint. Figure 2(a) illustrates the input-output relationships in the first measurement when the sample weight is given to the G_1 -system. In other words, the sample weight is put on the left goods-plate of a balance. Figure 2(b) does those relationships for the second measurement.

The knowledge of mechanics teaches us that the input to the $m \cdot G_1$ system or the $M \cdot G_2$ system is acceleration of gravity and the output is the moment of force applied to the left or right arm of the beam, respectively. Accordingly, we can write $y_1(t) = u_1(t)$ and $y_2(t) = u_2(t)$ in equilibrium, thus we get Eqs. (1a) and (1b). Elimination of G_1 and G_2 from Eqs.(1a) and (1b) yields the desired formula (2) which determines the mass value m .

Since the resulting formula (2) is independent in length of left and right arms of balance, this follows that static characteristics of two linear measuring systems, G_1 and G_2 , does not make any influence on the resulting value in case of input level measurement introducing the concept of double weighing procedure.

Consequently, it is quite natural to reach that the concept could be equally applied to the dynamic compensation of two linear dynamic measuring systems used under a conventional

static or specific dynamic condition.

3. Extended Version- Principle of the Double Measuring Method

3.1 Basic Idea of the Double Measuring Method

Let us consider the following measuring situation to extend the original double weighing procedure to dynamic measurement problems.

We prepare a pair of linear measuring devices whose transfer functions are $G_a(s)$ and $G_b(s)$, and a standard sensor M whose transfer function $M(s)$ is known. Here we assume the case where $G_a(s)$ and $G_b(s)$ are nearly the same but unknown. We then consider the problem to determine the dynamical characteristics of an object sensor m whose transfer function $m(s)$ is unknown through measurement of the transfer function ratio of $m(s)$ to $M(s)$ [1].

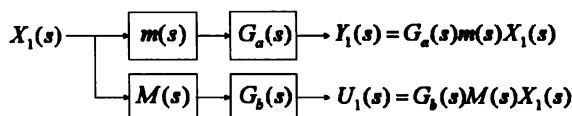


Fig. 3(a) Measurement 1

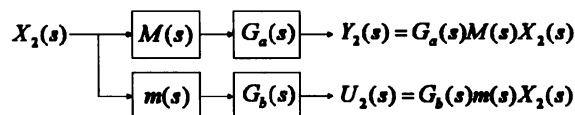


Fig. 3(b) Measurement 2

Measurement is conducted in the following steps:

1) First measurement:

After connecting respectively the object sensor m to measuring device G_a and the standard sensor M to measuring device G_b , the first measurement is carried out under a suitable input excitation, $x_1(t)$, and the output signal $y_1(t)$ and $u_1(t)$ are simultaneously recorded respectively by the corresponding measuring instruments. As an input-output relationship we have then the following relation:

$$Y_1(s) = G_a(s)m(s)X_1(s), \quad \dots(3)$$

and

$$U_1(s) = G_b(s)M(s)X_1(s). \quad \dots(4)$$

Elimination of $X_1(s)$ from Eqs.(3) and (4) then gives us the following relation.

$$\frac{Y_1(s)}{U_1(s)} = \frac{G_a(s)m(s)}{G_b(s)M(s)} = \frac{G_a(s)}{G_b(s)} \cdot \frac{m(s)}{M(s)}, \quad \dots(5)$$

where $Y_1(s)$ and $U_1(s)$ are Laplace transforms of $y_1(t)$ and $u_1(t)$, respectively.

2) Second measurement:

Exchanging the connecting positions of the standard device M and the object device m in the measuring systems, the output signals, $y_2(t)$ and $u_2(t)$, are recorded under another suitable input excitation, $x_2(t)$.

Similarly, we have the following relation:

$$\frac{Y_2(s)}{U_2(s)} = \frac{G_a(s)M(s)}{G_b(s)m(s)} = \frac{G_a(s)}{G_b(s)} \cdot \frac{M(s)}{m(s)}, \quad \dots(6)$$

where $Y_2(s)$ and $U_2(s)$ are Laplace transforms of $y_2(t)$ and $u_2(t)$, respectively.

Elimination of $G_a(s)/G_b(s)$ from Eqs.(5) and (6) gives us

$$\frac{Y_1(s)U_2(s)}{U_1(s)Y_2(s)} = \left[\frac{m(s)}{M(s)} \right]^2. \quad \dots(7)$$

Consequently, it turns out from Eq.(7) that we can evaluate the dynamical characteristics of the object sensor m in terms of the transfer function ratio $m(s)/M(s)$ without any influence of measuring device dynamics $G_a(s)$ and $G_b(s)$.

Remarks 2: Let us explain a few mathematical aspects of the double measuring method. If we consider the transfer function ratio $m(s)/M(s)$ and $G_a(s)/G_b(s)$ as a single parameter, we understand that Eq.(3) includes two unknown parameters; $m(s)/M(s)$ and $G_a(s)/G_b(s)$. Therefore, one more equation is necessary to solve $m(s)/M(s)$ independently from measurement. Then, the second measurement is conducted to provide this second equation of Eq. (4).

3) Evaluation of the transfer function ratio $m(s)/M(s)$:

By applying the linear system identification method with these output data

stored in memory during the first and second measurements, we can obtain the square of transfer function of a virtual linear system, which is defined as follows:

$$F^2(s) = \frac{Y_{21}(s)}{Y_{11}(s)}, \quad \dots(8)$$

hence $Y_{11}(s)$ and $Y_{21}(s)$ are Laplace transforms of input and output signals of the virtual linear system, respectively defined as

$$Y_{21}(s) = Y_1(s)U_2(s), \quad Y_{11}(s) = U_1(s)Y_2(s). \quad \dots(9)$$

Since the virtual transfer characteristic $F^2(s)$ has once been obtained, the transfer function $F(s)$ of a relative compensation filter (calibration filter) can also be obtained as the square root of transfer function of the virtual linear system. Since we can easily implement the compensating filter $F(s)$ by means of analog or digital devices, it turns out that the unknown dynamics of the object sensor m is given in principle as follows:

$$m(s) = F(s)M(s). \quad \dots(10)$$

3.2 Improved Double Measuring Method

If it is possible to give the identical input excitation to the two measuring devices, we can replace the second measurement block diagram of Fig. 3 by that of Fig. 4.

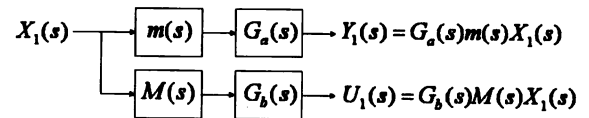


Fig. 4(a) Measurement 1

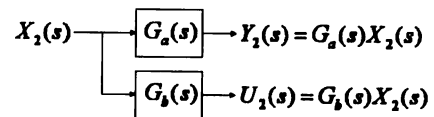


Fig. 4(b) Measurement 2

Accordingly, the improved version is stated as follows:

1) First measurement:

The measurement is the same as the first measurement of the double measuring method.

2) Second measurement:

Giving the identical input excitation directly to the two measuring devices $G_a(s)$ and $G_b(s)$, the output signals, $y_2(t)$ and $u_2(t)$, are recorded. We have then the following relation as the substitute for Eq.(6).

$$\frac{Y_2(s)}{U_2(s)} = \frac{G_a(s)}{G_b(s)} \quad \dots(11)$$

From Eqs. (5) and (11) we have the relation as follows:

$$\frac{Y_1(s)U_2(s)}{U_1(s)Y_2(s)} = \frac{m(s)}{M(s)} \quad \dots(12)$$

Consequently, we can obtain the required ratio $m(s)/M(s)$ in terms of Eq. (12).

This represents the fundamental relation of double measuring method, the extended version of Gauss' double weighing method, which is applicable to linear dynamic weighing or measurement processes.

4. Novel Method for Linear System Identification Independent of Sensor Dynamics

We consider the problem to identify the dynamics of a linear system T whose transfer function is $T(s)$. Let us assume the input and output signals have the same kind of physical quantity. We prepare a pair of linear measuring devices whose transfer functions are $G_a(s)$ and $G_b(s)$. for observation of the input and output signals, but $G_a(s)$ and $G_b(s)$. are assumed unknown.

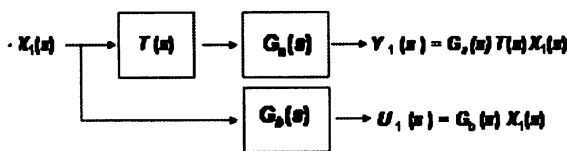


Fig. 5(a) Measurement 1

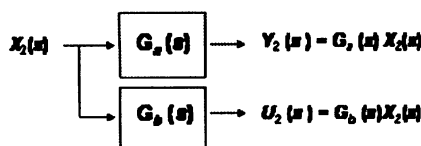


Fig. 5(b) Measurement 2

Figure 5 illustrates the block diagram of measurement by the novel method. Identification consists of two measuring processes; the first measurement and the second measurement.

1) First measurement:

The measurement is the same as that of conventional identification method. After connecting respectively the measuring devices G_a and G_b to the input and output of the system, the first measurement is carried out under a suitable input excitation, $X_1(s)$ and the input and output signals, $U_1(s)$ and $Y_1(s)$, are stored by the corresponding recorders.

As an input and output relationship, we have

$$\frac{Y_1(s)}{U_1(s)} = \frac{G_a(s)}{G_b(s)} \cdot T(s) \quad \dots(13)$$

Equation (13) implies a rigorous measurement equation in conventional identification processes. From Eq.(13), it is obvious that if we use the observed signals, $U_1(s)$ and $Y_1(s)$, to identify the dynamics of the system T , the identified result includes the measuring device dynamics $G_a(s)$ and $G_b(s)$ besides the system dynamics $T(s)$. That is to say, the order and parameter values of the mathematical model identified with the conventional method depend on the dynamical characteristics of the measuring devices used for signal observation to some extent.

2) Second measurement:

This measurement is correspond to the second measurement of the improved double measuring. So, we get the following relationship;

$$\frac{Y_2(s)}{U_2(s)} = \frac{G_a(s)}{G_b(s)} \quad \dots(14)$$

Accordingly, we can easily get the desired transfer characteristic $T(s)$ by solving a pair of Eqs. (13) and (14) as follows:

$$T(s) = \frac{Y_1(s)U_2(s)}{U_1(s)Y_2(s)} \quad \dots(15)$$

Equation (15) implies that we can obtain $T(s)$ without any influence of measuring dynamics

$G_a(s)$ and $G_b(s)$. This also suggests us a potential application of the improved double measuring method to linear system identification problems.

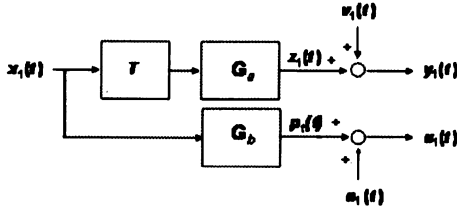


Fig. 6(a) Measurement 1

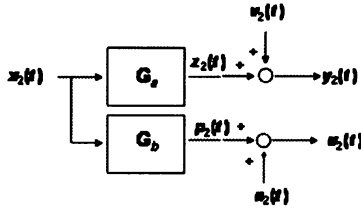


Fig. 6(b) Measurement 2

5. Measurement Noise Influence on the Identification

The identification procedure due to the improved double measuring method succeeds under the noise free condition. However, in the case when the measurement noise $n_i(t)$ and $v_i(t)$ ($i=1,2$) cannot be ignored in Fig. 5, only the signals $u_i(t)$, $y_i(t)$ are affected by the noise are observed, where it is assumed that the noise $n_i(t)$, $y_i(t)$ ($i=1,2$) are stationary time series with zero mean and they are uncorrelated to each other. If the signals $u_i(t)$ and $y_i(t)$ ($i=1,2$) are used in stead of $p_i(t)$ and $z_i(t)$, respectively, for the identification due to the improved double measuring method, then the result

$$\begin{aligned} \tilde{T}(s) &= \frac{Y_1(s)U_2(s)}{U_1(s)Y_2(s)} \\ &= \frac{Z_1(s) + V_1(s)}{P_1(s) + N_1(s)} \cdot \frac{P_2(s) + N_2(s)}{Z_2(s) + V_2(s)}, \quad \dots(16) \end{aligned}$$

is not expected to coincide with $T(s)$.

Since the noise $n_i(t)$ and $v_i(t)$ ($i=1,2$) are uncorrelated to each other, the Fourier transformation of the cross-covariance for each of the signals and $u_i(t)$ in measurement i ($i=1,2$) yields

$$S_{y_i u_i}(\omega) = S_{z_i p_i}(\omega), S_{u_i u_i}(\omega) = S_{p_i p_i}(\omega) + S_{n_i n_i}(\omega), \dots(17)$$

($i=1,2$)

where $S_{y_i u_i}(\omega)$ is the cross power spectral density with y_i and u_i and $S_{u_i u_i}(\omega)$ is the power spectral density of u_i . On the other hand, the cross spectral densities, the spectral densities and the frequency transfer functions satisfy

$$\left. \begin{aligned} \frac{S_{z_1 p_1}(\omega)}{S_{p_1 p_1}(\omega)} &= \frac{G_a(j\omega)T(j\omega)}{G_b(j\omega)} \\ \frac{S_{z_2 p_2}(\omega)}{S_{p_2 p_2}(\omega)} &= \frac{G_a(j\omega)}{G_b(j\omega)} \end{aligned} \right\}, \quad \dots(18)$$

From Eqs.(16) and (17), the system T is expressed by the cross spectral and spectral densities:

$$\begin{aligned} T(j\omega) &= \frac{S_{z_1 p_1}(\omega)S_{p_2 p_2}(\omega)}{S_{p_1 p_1}(\omega)S_{z_2 p_2}(\omega)} \\ &= \frac{S_{y_1 u_1}(\omega)}{S_{u_1 u_1}(\omega)} \cdot \frac{S_{u_2 u_2}(\omega)}{S_{y_2 u_2}(\omega)} \cdot \frac{1 + \{S_{p_1 p_1}(\omega)/S_{n_1 n_1}(\omega)\}^{-1}}{1 + \{S_{p_2 p_2}(\omega)/S_{n_2 n_2}(\omega)\}^{-1}}, \dots(19) \end{aligned}$$

where $S_{p_i p_i}(\omega)/S_{n_i n_i}(\omega)$ denotes the signal-to noise (SN) ratio at the observed output from the device G_b . Therefore, the system T can be identified from the observable signals u_i , y_i and SN ratios. That is, the measurement noise influence on the identified result appears in the last multiplied term;

$$\frac{1 + \{S_{p_1 p_1}(\omega)/S_{n_1 n_1}(\omega)\}^{-1}}{1 + \{S_{p_2 p_2}(\omega)/S_{n_2 n_2}(\omega)\}^{-1}}.$$

Since the SN ratios $S_{p_i p_i}(\omega)/S_{n_i n_i}(\omega)$ ($i=1,2$) are determined by the output channel of the device G_b , they are approximately regarded as equivalent. That is, the system T can be approximately be expressed by

$$T \cong \frac{S_{y_1 u_1}(\omega)}{S_{u_1 u_1}(\omega)} \cdot \frac{S_{u_2 u_2}(\omega)}{S_{y_2 u_2}(\omega)}. \quad \dots(20)$$

Moreover, $S_{y_i u_i}(\omega)/S_{u_i u_i}(\omega)$ in Eq.(20) is considered as the frequency transfer function of the system whose input is u_i and whose output is y_i . Thus, the system T is approximately equal to \tilde{T} of Eq.(16), which is identified result by the improved double

measuring method. This means that the noise influence on the result identified by the improved double measuring method can be removed by the principle of the method that two measurements 1 and 2 are required for the same measurement devices.

Remarks 3: In measurements 1 and 2, the input signals $x_1(t)$, $x_2(t)$ are not necessarily the same. Especially, when these signals $x_1(t)$, $x_2(t)$ are generated from the same stochastic source or the statistical properties of these signals are the same, SN ratios $S_{p1p1}(\omega)/S_{n1n1}(\omega)$, and $S_{p2p2}(\omega)/S_{n2n2}(\omega)$ coincide with each other and the equality of Eq.(19) holds. This means that the noise influence can be completely removed.

6. Feasibility Studies Through Simulation

In order to the validity of the improved double measuring method, we have considered a few numerical examples [3],[5]. The examples show for discrete time systems since the above discussion is also valid for diacrete time systems though the previous sections dealt with continuous time systems. Referring to Fig. 5, we set up a discrete-time simulation model as follows:

$$T(z) = \frac{0.314330 + 0.012327z^{-1} - 0.302003z^{-2}}{1 - 1.913713z^{-1} + 0.938367z^{-2}}, \dots(21)$$

and

$$\left. \begin{aligned} G_a(z) &= \frac{0.909091 + 0.909091 z^{-1}}{1 - 0.818182 z^{-1}} \\ G_b(z) &= \frac{1.7 + 1.7 z^{-1}}{1 - 0.666667 z^{-1}} \end{aligned} \right\}, \dots(22)$$

where $T(z)$, $G_a(z)$ and $G_b(z)$ are pulse transfer functions of T , G_a and G_b , respectively.

Substituting $z = \exp(j\omega T_s)$ into the above equations, we can calculate the frequency responses of G_a , G_b and T , where j is imaginary unit, ω is angular frequency and T_s is the sampling period (0.02 s). The results are shown in Figs. 7 and 8.

The first and second measurements are conducted respectively under the input excitations of white Gaussian noise sequences with mean 0 and variance 1. The set of output signals, $\{y_1(i), u_1(i)\}$ and $\{y_2(i), u_2(i)\}$

($i = 0, 1, \dots, 399$), was used to calculate $y(i)$ and $u(i)$. That is to say,

$$\left. \begin{aligned} y(i) &= y_1(i) * u_2(i) = \sum_{k=0}^{399} y_1(k) u_2(i-k) \\ u(i) &= u_1(i) * y_2(i) = \sum_{k=0}^{399} u_1(k) y_2(i-k) \end{aligned} \right\} \dots(23)$$

($i = 0, 1, \dots, 798$).

Based on the observed data $y_1(i)$ and $u_1(i)$ of the first measurement and the calculated data $y(i)$ and $u(i)$, the parameters of pulse transfer function Eq.(24) were determined with least square method [6].

$$\tilde{T}(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + \dots + b_n z^{-n}}{1 + a_1 z^{-1} + a_2 z^{-2} + \dots + a_n z^{-n}}, \dots(24)$$

where the order n was determined with Akaike's Information Criterion (AIC)[7].

The frequency responses based on the simulated results are shown in Fig. 8. Fig. 8 indicates that the simulated results with the conventional method is influenced by $G_a(z)$ and $G_b(z)$, but that the influence is eliminated by the improved method.

The measurement noise influence on the identified is also examined by simulation. Referring to Fig. 6, we set up input signals $x_1(t)$, $x_2(t)$ are generated as white Gaussian time series with mean 0 and variance 1, and measurement noise $n(i)$ and $v(i)$ ($i=1,2$) are generated as white Gaussian time series with mean 0 and variances 0.03 and 0.01, respectively.

For these systems, numerical simulation of measurements 1 and 2 are achieved and the system T is identified. That is, in measurement i ($i=1,2$), the frequency response function $S_{yiu}(\omega)/S_{uiu}(\omega)$ is identified as an ARX model whose input and output are u_i and y_i , respectively. The identified results are substituted for Eq.(20), and that yields the model identified by the improved double measuring method.

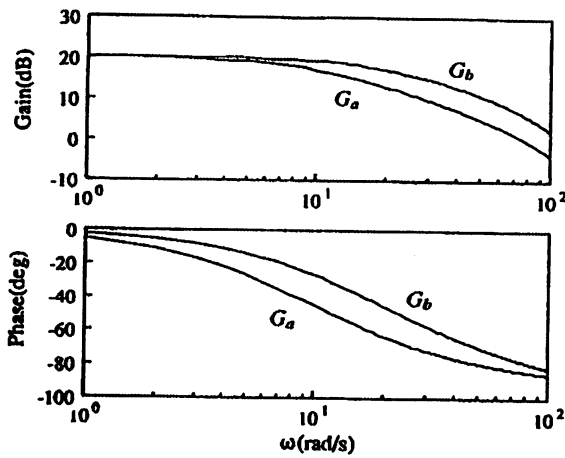


Fig. 7 Frequency responses of G_a and G_b

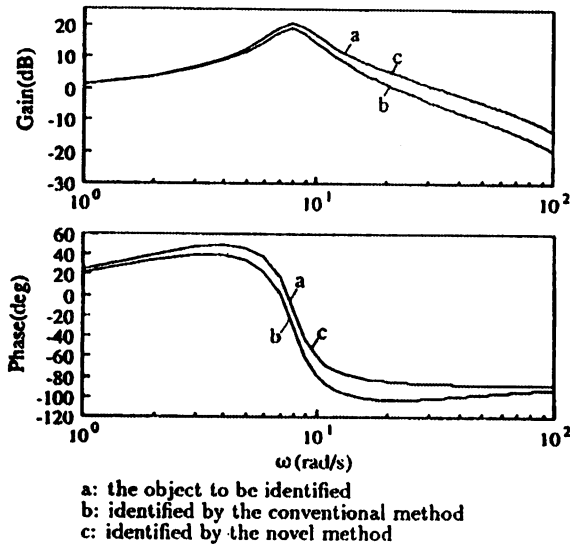


Fig. 8 Frequency responses of T with identified results

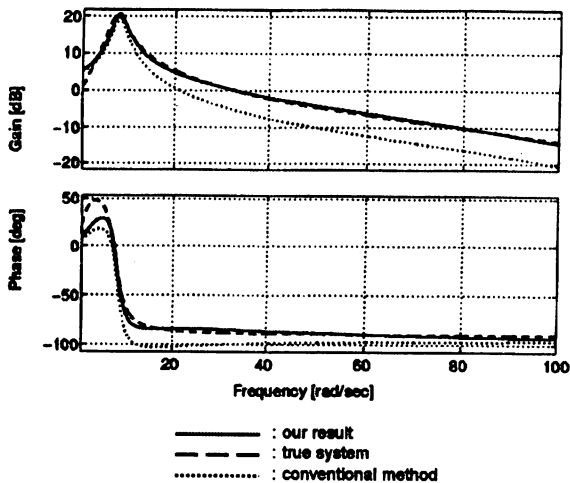


Fig. 9 Frequency responses

The frequency response of the identified model is plotted in Fig. 9 as the solid line. The dashed line indicate that of the true system T , and the dotted line indicates that of the model identified by the conventional method. Here, the conventional method means the identification method by which the influence of the measuring devices is ignored. That is, the model identified only by using data u_1 and y_1 of measurement 1 is regarded as the model identified by the conventional method. Since the data affected by the measuring devices G_a and G_b as well as noise $n_i(t)$ and $z_i(t)$, the identified result by the conventional method also includes the difference in dynamics between these devices and the influence of noise. This fact can be seen in Fig. 9, where the dotted line is apart from the others. While Fig. 8 shows that the response of the model identified by the improved double measuring method is close to the true one. This implies that the identification method by the double measuring method removes the influence of the difference in dynamics between two measuring devices as well as the measurement noise. Therefore, the improved double measuring is useful even if there exists measurement noise.

7. Conclusion

The relative compensation of differences in dynamics among plural measuring devices is significant for precise measurement. The dynamic multiple measuring method has been developed and applied for the relative compensation by the author and his research group [1]~[5].

In this tutorial paper, the author has described the basic concept of the dynamic multiple measuring method, the extended version of the Gauss' double weighing method, and has presented the analytical result on the measurement noise influence and a few results of numerical feasibility studies are given to suggest the potential application of this method to practical systems. Here, the analysis is paid to only the case when the improved double measuring method is applied

to the system identification problem, but the similar discussion is available for application of the double measuring method or the improved double measuring method to calibration problems and control problems.

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