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Characteristics of Fluid Flow of Froth on a Perforated Plate

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This paper represents a theoretical approach for estimating axial gas (liquid) holdup and mean gas (liquid) holdup in the froth (gas-liquid mixture formed by blowing gas into shallow liquid) on a perforated plate by means of the calculus of variation. The paper also describes some fundamental characteristics such as gas (liquid) holdup and/or liquid circulation in the froth, which are of importance to the performance of gas-liquid contactors. Of particular emphases for characteristics of the froth are axial and radial profiles of gas (liquid) holdup, mean gas (liquid) holdup and liquid circulation obtained experimentally.

Introduction

The operation of greatest importance for gas-liquid contactors such as bubble columns and/or plate columns, in which perforated plates are commonly used, is the dispersion of gas into shallow liquid. When gas is dispersed into liquid on a perforated plate, gas-liquid mixture, namely, froth appears. To clarify the behavior and/or characteristics of the froth is very difficult because the fluid dynamics of the system is extremely complex and poorly understood. Furthermore, in the application to chemical, petrochemical or biochemical reactions, mass transfer involving gaseous reactants occurs through the gas-liquid interface. Hence, gas (liquid) holdup and liquid circulation in the froth on a perforated plate are of great interest because of their direct effects on column efficiency. Therefore, to fully understand or predict the flow behavior or reactor performance, effects of gas (liquid) holdup and liquid circulation in the froth need to be accounted.

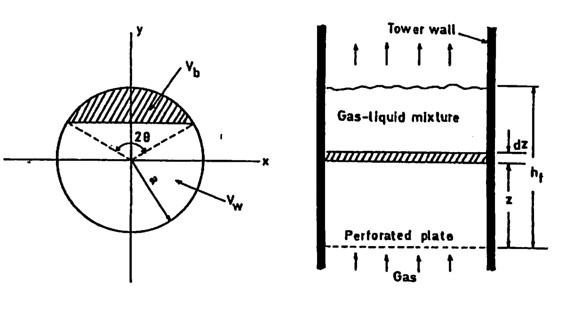
In this paper, a theoretical approach for estimating axial gas (liquid) holdup and mean gas (liquid) holdup in the froth on a perforated plate is presented based on calculus of variation. In addition, some important characteristics pertaining to gas (liquid) holdup and liquid circulation obtained experimentally are briefly shown. Of particular emphases are axial and radial profiles of gas (liquid) holdup, mean gas (liquid) holdup and liquid velocity in the froth.

1. Axial and Mean Gas (Liquid) Holdup

1.1 Theoretical Treatment

Azbel (1962) and Kim (1965) have made modeling attempts for axial and mean gas holdup in the froth on a perforated plate. While Azbel assumed a spherical bubble and Kim an elliptical bubble for gas holdup evaluation, they did not consider the effects of near wake of bubble. In the following, fundamental framework for a model based on the assumption of a spherical bubble including the near wake is presented to predict the axial gas holdup profiles and the mean gas holdup. Here the following assumptions are made to simplify the model formulation.

1) The bubble is a spherical cap bubble.
2) The energy of mixing and the energy of diffusion between liquid and gas are neglected.
3) The friction between gas and liquid is neglected.
4) The distribution of the gas holdup is uniform in the radial direction. This is not always true in practice.



(a) Structure of bubble (b) Elementary layer in froth

Fig. 1 Structure of bubble and elementary layer in froth

As shown in **Fig. 1(a)**, the spherical cap bubble is accompanied by liquid volume V_w as the wake (Collins, 1965a, 1965b, 1966, 1967). The whole volume containing both the bubble volume V_b and the wake volume V_w is approximated as a sphere with radius a. Davies and Taylor (1950) showed that θ =60 degree. As shown in **Fig. 1(b)**, let us consider, in the froth on a plate, an elementary layer with the height dz. Therefore, the gas holdup in this layer is

$$\varepsilon_G = \pi a^3\left(\frac{2}{3} - \cos\theta + \frac{1}{3}\cos^3\theta\right)n \qquad (1)$$

Where n is the number of bubbles in the unit volume. The potential energy of this layer is

$$dE_p = \left(1 - \varepsilon_G\right)\rho_l gzdz \qquad (2)$$

If the bubble accompanies liquid K times as much as the volume of the sphere with radius a, and the volume V_w, one bubble is accompanied by liquid of N_l times as much as the bubble volume

$$N_l = \frac{\frac{2}{3} + \cos\theta - \frac{1}{3}\cos^3\theta + \frac{4}{3}K}{\frac{2}{3} - \cos\theta + \frac{1}{3}\cos^3\theta} \qquad (3)$$

On the assumption that the kinetic energy of this elementary layer dE_k, is the sum of both the kinetic energy of the gas and the energy of the liquid accompanied by the bubble, and that the gas and liquid move at the same velocity, the following equation is obtained.

$$\begin{aligned} dE_k &= \frac{1}{2}\rho_g U_b^2 \pi a^3\left(\frac{2}{3} - \cos\theta + \frac{1}{3}\cos^3\theta\right)ndz \\ &+ \frac{1}{2}\rho_l U_b^2 \pi a^3\left(\frac{2}{3} - \cos\theta + \frac{1}{3}\cos^3\theta\right)nN_l dz \\ &= \left(\rho_l + N_l\rho_l\right)\varepsilon_G U_b^2 dz/2 \end{aligned} \qquad (4)$$

The surface area of the spherical cap bubble is expressed as follows

$$S = \pi a^2\left(\sin^2\theta + 2 - 2\cos\theta\right) \qquad (5)$$

Then the energy of the surface tension dE_s is given by

$$dE_s = \pi a^2\left(\sin^2\theta + 2 - 2\cos\theta\right)\sigma ndz \qquad (6)$$

If the flow around the sphere containing the bubble and the wake can be assumed to be the potential flow, K=1/2 (Darwin, 1953; Milne-Thomson, 1968). Therefore

$$dE_k = \frac{1}{2}\left(\rho_g + \frac{43}{5}\rho_l\right)U_{gc}^2\frac{dz}{\varepsilon_G} \qquad (7)$$

$$\varepsilon_G = \frac{5}{24}\pi a^3 n \qquad (8)$$

$$dE_s = \frac{7}{4}\pi a^2\sigma ndz = \frac{42}{5a}\sigma\varepsilon_G dz \qquad (9)$$

The total energy of this elementary layer E_t is

$$E_t = \int_0^{h_f}\left\{\left(1 - \varepsilon_G\right)\rho_l gz + \left(\rho_g + \frac{43}{5}\rho_l\right)\frac{U_{gc}^2}{2\varepsilon_G} + \frac{42}{5a}\sigma\varepsilon_G\right\}dz \qquad (10)$$

The system should be stable when E_t takes a minimum value under the subsidiary condition that the liquid holdup is constant

$$H = \int_0^{h_f}\left(1 - \varepsilon_G\right)dz = \text{const.} \qquad (11)$$

and the boundary condition

$$\varepsilon_G\left(h_f\right) = 1 \qquad (12)$$

Therefore

$$E_t\left(\varepsilon_G\right) = \int_0^{h_f} F\left(\varepsilon_G,\ z,\ U_{gc},\ \rho_l,\ \sigma\right)dz \qquad (13)$$

$$H\left(\varepsilon_G\right) = \int_0^{h_f} G\left(\varepsilon_G\right)dz \qquad (14)$$

After introducing the Lagrange multiplier λ, the following Euler's equation is obtained by using the calculus of variation.

$$\frac{\partial F}{\partial\varepsilon_G} + \lambda\frac{\partial G}{\partial\varepsilon_G} = 0 \qquad (15)$$

Substitution of the function F and G into Eq. (15) gives

$$\varepsilon_G = \sqrt{\frac{\left(\rho_g + \frac{43}{5}\rho_l\right)U_{gc}^2}{2\left(\frac{42}{5a}\sigma - \lambda - \rho_l gz\right)}} \qquad (16)$$

Further, substitution of Eq. (12) into Eq. (16) gives

$$\lambda = \frac{42}{5a}\sigma - \rho_l gh_f - \frac{1}{2}\left(\rho_g + \frac{43}{5}\rho_l\right)U_{gc}^2 \qquad (17)$$

From Eqs. (11) and (16), substituting the value of λ and regarding $\rho_l >> \rho_g$, we can get the froth height h_f as follows

$$h_f = H\left(1+\sqrt{\frac{86}{5}Fr}\right) \quad (18)$$

where $Fr = U_{gc}^2/(gH)$. Therefore, the axial gas holdup is

$$\varepsilon_G = \sqrt{\frac{\frac{43}{10}Fr}{\left(\sqrt{\frac{86}{5}Fr}+\frac{43}{10}Fr+1\right) - z/H}} \quad (19)$$

From Eq. (18), the mean gas holdup is obtained

$$\overline{\varepsilon_G} = \frac{\sqrt{\frac{86}{5}Fr}}{1+\sqrt{\frac{86}{5}Fr}} \quad (20)$$

Therefore, the mean gas-liquid holdup ratio is (Takahashi *et al.*, 1973)

$$\frac{\overline{\varepsilon_G}}{1-\overline{\varepsilon_G}} = \sqrt{\frac{86}{5}Fr} \quad (21)$$

1.2 Axial Profiles of Gas (Liquid) Holdup

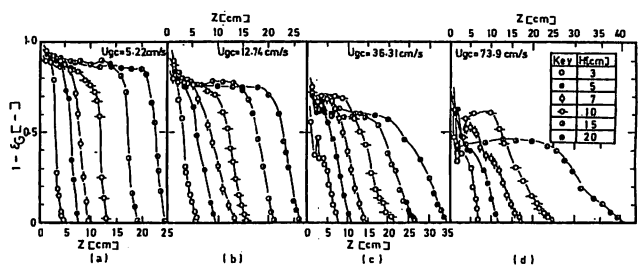


Fig. 2 Axial profiles of mean liquid holdup

Figure 2, obtained by Takahashi *et al.* (1974), shows axial profiles of the mean liquid holdup on a perforated $\varepsilon_l (= 1 - \varepsilon_G)$ measured by means of γ ray technique. From these figures, ε_l decreases suddenly near the surface of a perforated plate (bubble formation region); then the degree of the diminution becomes mild (middle region); and above this region, ε_l (=1-ε_G) decreases (upper layer region) with increasing gas velocity. At high gas velocity, the minimum value of ε_l appears at about 2-3 cm from a sieve plate. This is probably due to stagnation of the disrupt bubbles caused by the gas jet near the sieve plate. The middle region has a tendency to show a constant value of ε_l at high liquid depth. This is most likely due to the fact that the bubble rises at the terminal velocity balanced with the buoyancy, the drag force and the inertial force in this region. The usual bubble column with a deep pool of liquid may correspond to the contactor with large middle region.

Figure 3, presented by Takahashi *et al.* (1974), shows comparison of the calculated values from Eq. (19) with the experimental ones of ε_G at U_{gc}=6.82 cm/s and various liquid depths. At low liquid depth, the measurements agree well with the predicted ones comparatively. At large liquid depth, however, the values of middle region are nearly constant, and the predicted value is a little large. As mentioned above, this may be attributed to the terminal velocity of bubble in this region. In Fig. 3, the dashed lines show the results obtained by Azbel who did not take into account the effect of the near wake of bubble.

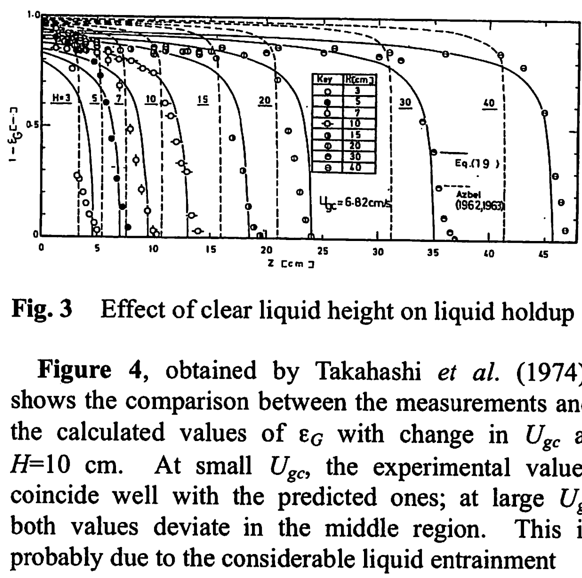


Fig. 3 Effect of clear liquid height on liquid holdup

Figure 4, obtained by Takahashi *et al.* (1974), shows the comparison between the measurements and the calculated values of ε_G with change in U_{gc} at H=10 cm. At small U_{gc}, the experimental values coincide well with the predicted ones; at large U_{gc} both values deviate in the middle region. This is probably due to the considerable liquid entrainment

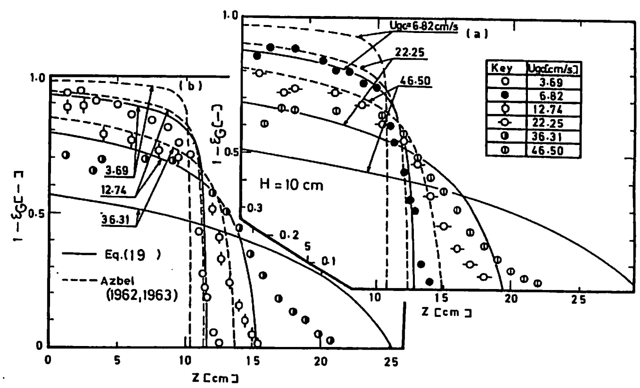


Fig. 4 Effect of gas velocity on liquid holdup

from the top of the froth and the violent disturbance in the upper layer region.

1.3 Mean Gas-Liquid Holdup Ratio

As shown in **Fig. 5** including the results of Braulick *et al.* (1965) and Yoshitome (1963), by Takahashi *et al.* (1974), the values of the mean gas-liquid holdup ratio, in the range of H=3-70 cm, are plotted against Froude number Fr depending on Eq.(21). At large Fr, both the theoretical and experimental values are in good agreement. In this figure, the dashed line is the result obtained by Azbel and the dotted line the result by Kim.

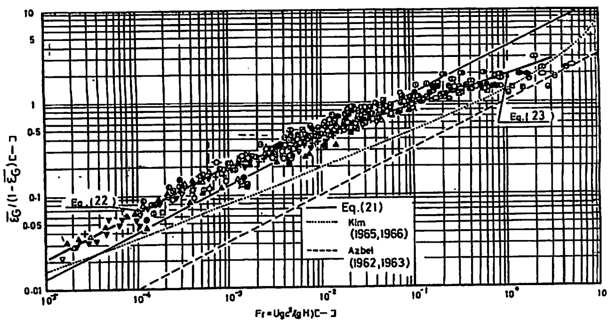


Fig. 5 Gas liquid holdup ratio versus Froude number

From Fig.5, it is obvious that the theoretical equation of the mean gas-liquid holdup ratio Eq.(21) agrees with the experimental values better than do those obtained by Azbel and Kim. Also Eq.(21) does not always represent the measurements. Therefore, by consideration of these measurements, empirical equations were obtained as follows

$$\frac{\bar{\varepsilon}_G}{1-\bar{\varepsilon}_G} = 6.5\sqrt{Fr} \qquad Fr \le 8.5 \times 10^{-4} \qquad (22)$$

$$\frac{\bar{\varepsilon}_G}{1-\bar{\varepsilon}_G} = 2\sqrt[3]{Fr} \qquad Fr \ge 8.5 \times 10^{-4} \qquad (23)$$

2. Radial Profiles of Gas Holdup and Liquid Circulation

Hills (1974) and Katoh *et al.* (1975) showed that radial profiles of gas holdup and liquid circulation velocity do not vary along the axial direction except in the vicinity of the plate and at the top of the froth. **Figure 6**, obtained by Miyahara *et al.* (1989), shows radial profiles of gas holdup measured by means of a conductivity probe and void fraction unit and liquid velocity measured by means of the Pavlov tube described previously by Hills (1974). As illustrated, gas holdup shows a maximum at a certain radial position; a behavior not commonly found in the usual bubble column with a deep pool of liquid. Moreover, profiles of liquid velocity are related to those of gas holdup, in that the local liquid velocity shows a maximum at the same position as that at which maximum gas holdup occurs.

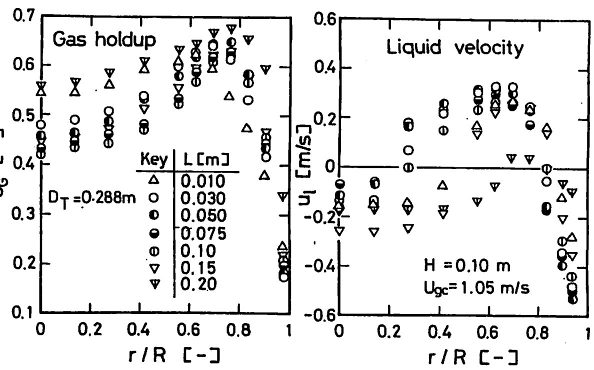


Fig. 6 Profiles of gas holdup and liquid velocity

Axial classification of the froth regime as presented by Miyahara *et al.* (1989) is illustrated in **Fig. 7**. Here the gas holdup at the center is plotted as a function of the ratio of axial position to liquid depth. As seen from this figure, the froth regime can be divided into three regions along the axial direction as described above: bubble formation region for L/H<0.3, middle region for 0.3<L/H<1.6 and upper layer region for L/H>1.6. The profiles of gas holdup and liquid velocity in the middle region show a distinctive behavior in that they have a maximum at a certain radial position. Therefore, the following concerns only the middle region.

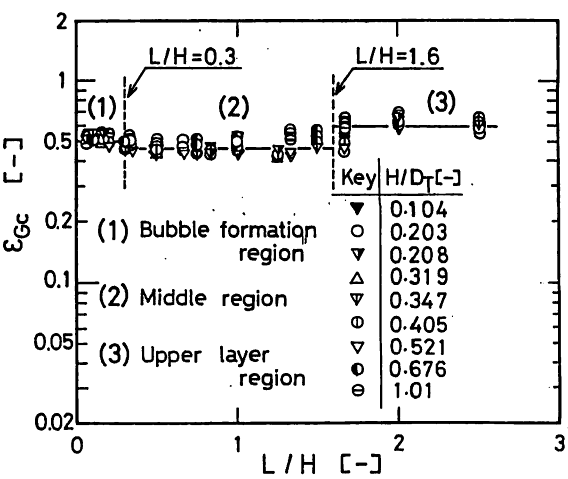


Fig. 7 Froth characteristics along axial direction

Figure 8, obtained by Miyahara *et al.* (1989), shows radial profiles of gas holdup and liquid velocity as a parameter of superficial gas velocity in the middle region. Although of course the parameter is different, **Fig. 9** shows radial profiles of gas holdup and liquid velocity. From these figures,

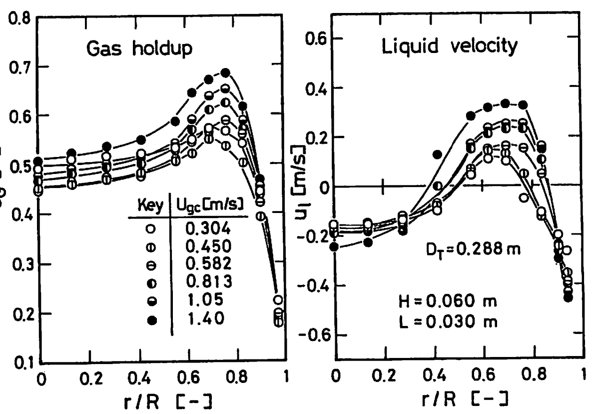


Fig. 8 Effect of superficial gas velocity on profiles of gas holdup and liquid velocity

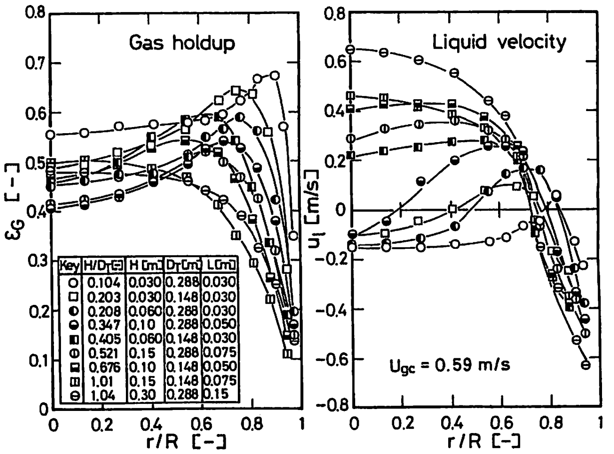


Fig. 9 Effect of liquid depth to column diameter ratio on profiles of gas holdup and liquid velocity

radial positions showing a maximum in gas holdup and upward liquid velocity move towards the column center with increasing liquid depth and decreasing column diameter, namely as H/D_T increases. At higher H/D_T, there is a central maximum in both gas holdup and liquid velocity; these phenomena are commonly noticed for the usual bubble column with a deep pool of liquid. The H/D_T value at the transition from the plate column to the bubble column regime is deduced to be around unity.

Hills (1974) and Miyauchi *et al.* (1970) studied radial gas holdup in bubble column theoretically using the theory of circulation. However, this theory requires the use of some unknown variables. Therefore, Miyahara *et al.* (1989) tried to correlate profiles of gas holdup in the middle froth region by the following equations.

$$\frac{\varepsilon_{Gm}-\varepsilon_G}{\varepsilon_{Gm}} = \left(\frac{r-r_o}{R-r_o}\right)^n \qquad r \ge r_o \tag{24}$$

$$\frac{(\varepsilon_G-\varepsilon_{Gc})\varepsilon_{Gm}}{(\varepsilon_{Gm}-\varepsilon_{Gc})\varepsilon_G} = \left(\frac{r}{r_o}\right)^{n'} \qquad r \le r_o \tag{25}$$

where r_o, ε_{GC}, ε_{Gm}, n and n' are given as follows

$$\left(\frac{r_o}{R}\right)\left(\frac{H}{D_T}\right)^{0.2} = 0.55Fr^{0.06} \tag{26}$$

$$\varepsilon_{Gc} = 0.46 \tag{27}$$

$$\frac{\varepsilon_{Gm}-\varepsilon_{Gc}}{\varepsilon_{Gm}} = 0.3 - 0.053Fr^{-0.82} \tag{28}$$

$$n = 0.3Fr^{-1} + 1.8Fr^{-0.05} \tag{29}$$

$$n' = 0.78Fr^{0.08}\left(\frac{H}{D_T}\right)^{-0.5} \tag{30}$$

Figure 10, obtained by Miyahara *et al.* (1989), shows a comparison of results calculated using Eqs. (24) and (25) with the experimental measurements. The calculated curves compare favorably with the measurements except for those where H/D_T=1.01. For the sake of comparison, the empirical results of Katoh *et al.* (1975) for bubble columns are shown in the same graph.

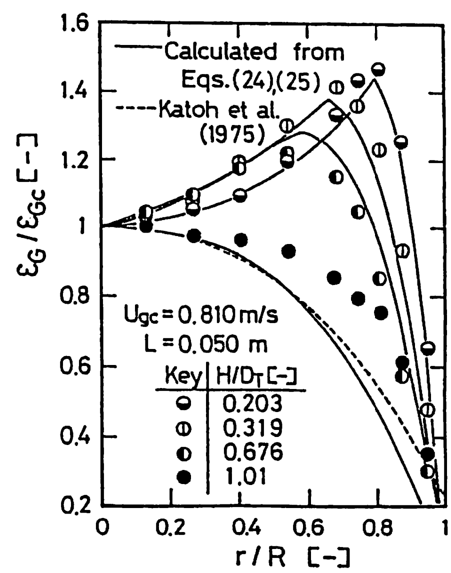


Fig. 10 Comparison between calculated and experimental measurements

Concluding Remarks

In perforated plate columns with gas bubbles as a discrete phase, the froth regime formed by blowing gas into shallow liquid on a perforated plate can be divided into three regions along the axial direction: bubble formation region, middle region and upper layer region. The axial profile of gas holdup based on theoretical analysis including the effect of the near wake of bubble taking into account the Euler's equation by means of the calculus of variation agrees with the measurements better than that obtained by Azbel. The mean gas holdup is better expressed by the theoretical equation derived from the axial profile of gas holdup than the results obtained by Azbel and Kim. However, the measurements behave slightly differently from the theoretical equation. Therefore, the empirical equations were obtained. As for the radial gas holdup and liquid circulation, there is a maximum in the gas holdup at a certain radial position with a central minimum unlike the behavior in the usual bubble column. This behavior corresponds to that of radial profiles of liquid velocity. Significant information on gas holdup and liquid circulation in the usual bubble column is available in the literature. More work is needed, however, to generalize the fluid flow dynamics of froth on a perforated plate.

Notation

a = radius of spherical cap bubble, m
D_T = column diameter, m
dE_k = kinetic energy of elementary layer per unit cross-sectional area, kg/s^2
dE_p = potential energy of elementary layer per unit cross-sectional area, kg/s^2
dE_s = energy of surface tension of elementary layer per unit cross-sectional area, kg/s^2
E_t = total energy per unit cross-sectional area, kg/s^2
F = function defined by Eq.(13), kg/(m.s^2)
Fr = Froude number $\{=U_{gc}^2/(gH)\}$
g = gravitational acceleration, m/s^2
G = function defined by Eq.(14), kg/(m.s^2)
H = liquid depth, m
h_f = froth height, m
L = measuring height, m
n = number of bubble in unit volume, m^{-3}
n = exponent in Eq.(29)
n' = exponent in Eq.(30)
N_l = constant defined by Eq.(3)
R = column radius, m
r = radial distance, m
r_o = r at maximum gas holdup, m
S = surface area of a spherical cap bubble, m^2
U_b = bubble rise velocity, m/s
U_{gc} = superficial gas velocity, m/s
u_l = upward liquid velocity, m/s
V_b = bubble volume, m^3
V_w = wake volume, m^3
z = distance from a perforated plate, m

ε_G = gas holdup
$\overline{\varepsilon_G}$ = mean gas holdup
ε_{Gc} = gas holdup at the center
ε_{Gm} = maximum gas holdup
ε_l = liquid holdup
θ = angle, rad
λ = Lagrange multiplier
ρ_g = gas density, kg/m^3
ρ_l = liquid density, kg/m^3
σ = surface tension, N/m

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