An Auto-Tuning Method of the Membership Functions of Class C^0.

V. Lachkhia

Department of Information and Computer Engineering

Okayama University of Science,

Ridai-cho 1-1, Okayama 700, Japan

(Received October 7, 1996)

In this paper, we propose an auto-tuning method of the membership functions of simplified fuzzy inference rules. The parameters of membership functions are optimized by method of steepest descent. The method was applied to the water filtration control system.

1. INTRODUCTION

Fuzzy logic control is currently a popular technique for engaging human expertise in process control applications. A fuzzy logic control strategy attempts to synthesize the linguistic control protocol of skilled human operators in a real-world environment. However, a fuzzy controller based directly on operator-specified or common sense rules often exhibits poor performance characteristics. This problem arises because the initial rules are often crude and have to be refined to achieve better performance.

Auto-tuning is a method which refines the fuzzy control policy by adjusting various controller parameters based on a set of performance measures. The important part of auto-tuning is adapting the membership functions. Refining the control policy by directly modifying the membership functions was studied in [8-11] .

In [8-9] H. Ichihashi introduce simplified fuzzy reasoning. The consequent parts of the simplified fuzzy reasoning are expressed by real values. In [8-9] the tuning of the consequent parts of the inference rules by method of steepest descent was proposed. This method was extended in [11] and the parameters of both antecedent and consequent parts of the simplified fuzzy reasoning were tuned at the same time. The membership functions of the antecedent parts in [11] were defined as equilateral triangles.

We extend methods of [9], [11] and allow use of membership functions of the arbitrary triangular and trapezoidal shapes. Based on an expert's selected training set, the parameters of both antecedent and consequent parts of the simplified fuzzy reasoning are tuned by method of steepest descent.

2. SIMPLIFIED FUZZY REASONING AND MEMBERSHIP FUNCTIONS FROM C^0

When input of the fuzzy control system is defined as $x_1, \ldots, x_m$ and output is y, the i -th rule of the simplified fuzzy reasoning is the following expression

$$\text{If } x_1 \text{ is } A_{i1} \text{ and } \ldots\ x_m \text{ is } A_{im} \text{ then } y \text{ is } w_i;\ (i = 1\ldots n)$$

where i is rule's number, $A_{i1}, \ldots, A_{im}$ are membership functions and w_i is the real value of the consequent part.

The output y of the simplified fuzzy reasoning is calculated as below

$$y = \frac{\sum_{i=1}^{n} \mu_i \cdot w_i}{\sum_{i=1}^{n} \mu_i} \quad (1) \qquad \mu_i = A_{i1}(x_1) \cdot A_{i2}(x_2) \cdot \ldots \cdot A_{im}(x_m) \quad (2)$$

Fuzzy membership functions can have different shapes, depending on the designer's preference or experience. In practice fuzzy engineers have found that triangular and trapezoidal shapes simplify computation and help capture the modeler's sense of fuzzy numbers.

Let us consider the membership functions of the arbitrary triangular and trapezoidal shapes. A triangular membership function A_{ij} is set (Fig. 1) by parameters a_{ij}, p_{ij}, q_{ij} ($q_{ij} > 0, q_{ij} > 0$). A trapezoidal membership function A_{ij} can be given (Fig.2) by parameters a_{ij}, q_{ij} or b_{ij}, p_{ij}, or can be given (Fig. 3) by parameters $a_{ij}, b_{ij}, q_{ij}, p_{ij}$ ($q_{ij} > 0, p_{ij} > 0$).

Trapezoidal membership functions from Fig. 2 are basic functions. All other membership functions of C^0 can be expressed by means of them. Let's denote membership functions from Fig. 2 as $A_{ij}^{a,q}(x_j)$ and $B_{ij}^{b,p}(x_j)$. Then we have

$$A_{ij}^{a,q}(x_j) = \frac{|x_j - a_{ij} - q_{ij}| - |x_j - a_{ij}|}{2 \cdot q_{ij}} + \frac{1}{2} \text{ and } B_{ij}^{b,p}(x_j) = \frac{|x_j - b_{ij} + p_{ij}| - |x_j - b_{ij}|}{2 \cdot p_{ij}} + \frac{1}{2}$$

For the triangular membership function A_{ij} (Fig. 1) we have

$$A_{ij}(x_j) = A_{ij}^{a,q}(x_j) + B_{ij}^{a,p}(x_j) - 1 \quad (3)$$

$$\text{so} \quad A_{ij}(x_j) = \frac{|x_j - a_{ij} - q_{ij}| - |x_j - a_{ij}|}{2 \cdot q_{ij}} + \frac{|x_j - a_{ij} + p_{ij}| - |x_j - a_{ij}|}{2 \cdot p_{ij}}$$

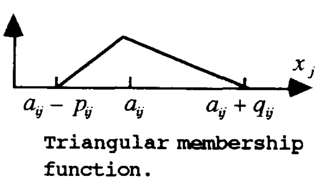

Fig. 1 Triangular membership function.

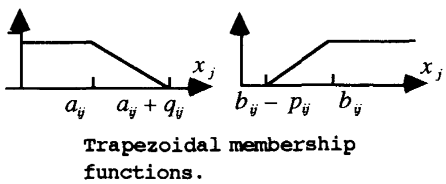

Fig. 2 Trapezoidal membership functions.

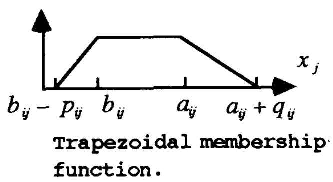

Fig. 3 Trapezoidal membership function.

For the trapezoidal membership function A_{ij} (Fig. 3) we have

$$A_{ij}(x_j) = A_{ij}^{a,q}(x_j) + B_{ij}^{b,p}(x_j) - 1 \tag{4}$$

so $$A_{ij}(x_j) = \frac{|x_j - a_{ij} - q_{ij}| - |x_j - a_{ij}|}{2\cdot q_{ij}} + \frac{|x_j - b_{ij} + p_{ij}| - |x_j - b_{ij}|}{2\cdot p_{ij}}$$

3. AUTO-TUNING ALGORITHM

The parameters of the membership functions and the real values of the consequent parts are tuned automatically by the method of steepest descent based on an expert's selected input-output data.

Let $x_1^l, \ldots, x_m^l, y^l (l = 1, \ldots, k)$ is the training set i.e. selected input-output data. Then $E = \frac{1}{2}\sum_{l=1}^{k}(y - y^l)^2$ expresses the error of the fuzzy control, where y is the actual value of the fuzzy control and y^l is the desired value. Minimizing E we find the optimal values of the parameters. The problem of minimizing the sum of squared errors is a classical one. We minimize performance function E by the steepest descent procedure.

Denote $f(z_1, \ldots, z_n)$ as $f(\tilde{z})$. Then the steepest descent method for minimizing f governed by the following equation

$$\tilde{z}(t+1) = \tilde{z}(t) - \alpha\cdot\nabla f\Big|_{\tilde{z}=\tilde{z}(t)} \tag{5}$$

in which $\tilde{z}(t)$ is the "old" value of $\tilde{z}$; $\tilde{z}(t+1)$ is the "new" value of $\tilde{z}$; ∇f is the gradient of f and coefficient α is a real number.

To simplify the calculation we use (5) to minimize $\frac{1}{2}(y - y^l)^2$. For the parameter of the consequent part w_i we get from (5) following tuning rule

$$w_i(t+1) = w_i(t) - C_w\cdot(y - y^l)\cdot\frac{\mu_i}{\sum_{i=1}^{n}\mu_i}$$

which is the same as in [9], [11]. For the parameters of the trapezoidal membership function $A_{ij}^{a,q}(x_j)$ we get from (5) following tuning rules

$$a_{ij}(t+1) = a_{ij}(t) - C_a\cdot(y - y^l)\cdot(w_i - y)\cdot B_{ij}\cdot\frac{\operatorname{sgn}(x_j - a_{ij}) - \operatorname{sgn}(x_j - a_{ij} - q_{ij})}{2\cdot q_{ij}} \tag{6}$$

$$q_{ij}(t+1) = q_{ij}(t) - C_q\cdot(y - y^l)\cdot(w_i - y)\cdot B_{ij}\cdot \frac{|x_j - a_{ij}| - |x_j - a_{ij} - q_{ij}| - q_{ij}\cdot\operatorname{sgn}(x_j - a_{ij} - q_{ij})}{2\cdot q_{ij}^2}$$

and for the parameters of the trapezoidal membership function $B_{ij}^{b,p}(x_j)$ we have

$$b_{ij}(t+1) = b_{ij}(t) - C_b\cdot(y - y^l)\cdot(w_i - y)\cdot B_{ij}\cdot\frac{\operatorname{sgn}(x_j - b_{ij}) - \operatorname{sgn}(x_j - b_{ij} + p_{ij})}{2\cdot p_{ij}} \tag{7}$$

$$p_{ij}(t+1) = p_{ij}(t) - C_p\cdot(y - y^l)\cdot(w_i - y)\cdot B_{ij}\cdot \frac{|x_j - b_{ij}| - |x_j - b_{ij} + p_{ij}| + p_{ij}\cdot\operatorname{sgn}(x_j - b_{ij} + p_{ij})}{2\cdot p_{ij}^2}$$

where $B_{ij} = \prod_{\substack{q=1 \\ q \neq j}}^{m} A_{iq}(x_q^l) \cdot (\sum_{i=1}^{n} \mu_i)^{-1}$ and C_w, C_a, C_b, C_p, C_q are the coefficients representing α in (5). For the parameters of the triangular (Fig. 1) and trapezoidal (Fig. 3) membership functions tuning rules can by easily calculated using (3), (4), (6) and (7).

The considered membership functions are from the class C^0. Therefore in the points where the partial derivatives do not exist we redefine them to equal 0. The result of the minimization of E considerably depends on the values of the coefficients C_w, C_a, C_b, C_p, C_q and the initial values of the tuning parameters.

There are an unlimited number of ways in which α in (5) can be selected. The case in which α becomes infinitesimal is very often in use. This avoids oscillating around the extreme and gives the convergent of the algorithm.

Another choice for α is "best-step steepest descent". Because of the computations involved in evaluating the gradient of $f(\tilde{z})$ at a given point, it is usually advantageous to make the most of each gradient computation before making another; that is, to search in the direction of the gradient until

$$\partial f(\tilde{z}(t) - \alpha \cdot \nabla f|_{\tilde{z}=\tilde{z}(t)}) / \partial \alpha = 0$$

The choice of the initial values of the parameters is an important one. We can use preliminary optimization techniques to find a good starting point. The "global" search methods as the nonsequential methods, or the pattern and the directed array search [13] can be used for this purpose. Another reasonable choice for the initial values are the expert defined membership functions.

A brief plan of the algorithm is as follow

Step 1. Set initial values of the tuning parameters

Step 2. Input training data $x_1^l, \ldots, x_m^l$ and using (1), (2) calculate y

Step 3. Using tuning rules and the values of y and y^l calculate the new values of the tuning parameters.

Step 4. Repeat step 2 and step 3 until the change of E becomes less then ε.

The described method can tune not only the parameters of the membership functions, but based only on the training set this method can also define set of the rules of the fuzzy control lake in [11] . But to get high performance it is probably necessary to use special algorithms for the rules tuning. In this paper we consider only tuning of the parameters of the membership functions. After obtaining the appropriate set of rules the described method can be useful for tuning membership functions.

4. APPLICATION TO THE WATER FILTRATION CONTROL SYSTEM

To demonstrate the usefulness of the method we use the above tuning algorithm for the fuzzy system which controls water purification process in water filtration plants. This fuzzy control system was developed in [7]. We use the same expert's defined inference rules and training set.

The control system for water purification described in [7] has 7 input variables.

There are TU1, ALK, TEMP, TUSE, FLOC, TUUP and STAT. The output of the fuzzy system is called DDOS. For detail consideration see [7]. The following inference rules were defined by the expert:

if TU1 is ST then DDOS is PM;
if TU1 is MM and TUSE is IL and TEMP is IS then DDOS is NM;
if TU1 is SA and ALK is LA and TEMP is SA then DDOS is PM;
if ALK is SA and TU1 is LA then DDOS is NM; (8)
if TUSE is LA then DDOS is PM; if TUUP is LA then DDOS is PB;
if TUUP is ML then DDOS is PM; if TUUP is MM then DDOS is PS;
if FLOC is SA then DDOS is PM; if STAT is LA then DDOS is PS;

where SA means small, MM means medium, LA means large, IL means inverse small, ML means medium large, PM means positive medium, NM means negative medium, PB means positive big, PS means positive small and ST means smaller than.

Table 1 (column TU1, ALK, TEMP, TUSE, FLOC, TUUP, STAT, DDOS) shows the expert's selected input-output data. The membership functions in [7] were chosen to make the error on the training set small. The compositional rule of fuzzy inference was used for the fuzzy output calculations in [7]. The membership functions defined in [7] are shown in Fig. 4 and Fig. 6. Fig. 4 describes the membership functions of the antecedent parts and Fig. 6 describes the membership functions of the consequent parts. Table 1 shows the error of the fuzzy control for the training data. The error was calculated as the difference between the actual value of the fuzzy control y and desired value y^l. The sum of the absolute values of the errors on the training set of the fuzzy control system defined in [7] is equal to 19.8.

Note that in some cases changes to the membership functions may not appreciably improve the control process. This is because the content of the rules is very important. Changes to the controller rules affects the control process more then changes to the membership functions. Therefore it is very important to choose the appropriate set of rules.

The set of rules in [7] is not so carefully chosen. Suppose that TUUP is MM and FLOC is SA. Then from the 8-th rule it follows that DDOS is PS, but the 9-th rule shows that DDOS is PM. So there are some contradictions in the rules of [7].

Despite the shortcomings of the rules we use the same 10 rules to turn the parameters of the membership functions. Our purpose is to minimize errors on the training set by tuning parameters of the membership functions.

The starting point is important for the steepest descent method. The initial values of the parameters of the antecedent parts in our algorithm were defined as in [7]. Thus membership functions of the antecedent parts described in [7] were chosen as the starting point. The real value of the consequent part $w_i (i = 1, \ldots, 10)$ was chosen to yield a maximum of the fuzzy membership function of DDOS described in i-th rule of (4). This is a reasonable starting point for iterative optimization. The values of the coefficients in the tuning rules ware taken as infinitesimal.

Table 1 Training set and errors of fuzzy control

TUI	ALK	TEMP	TUSE	TUUP	FLOC	STAT	DDOS	Error of fuzzy control in [1]	Error after tuning
5.0	13.7	11.5	0.4	0.0	0.5	1.0	1.7	−0.7	−0.9
5.0	13.7	11.5	0.5	0.0	0.5	0.0	1.8	−1.3	−0.6
5.0	13.9	11.6	0.5	0.0	0.5	0.0	1.8	−1.2	−0.3
5.0	13.7	11.7	0.4	0.0	0.5	0.0	1.8	−1.1	−0.4
6.0	13.4	11.7	0.5	0.0	0.5	0.0	1.6	−0.4	−0.0
19.0	14.2	11.0	0.4	0.3	0.3	1.0	0.5	0.5	0.0
28.0	13.8	10.7	0.7	0.5	0.3	0.0	0.2	−0.3	−0.2
32.0	13.8	10.5	0.9	1.0	0.3	0.0	4.7	1.3	1.0
33.0	13.8	10.5	0.9	1.0	0.3	0.0	4.7	0.2	0.0
33.0	13.8	10.6	0.9	1.0	0.3	0.0	4.6	−0.1	−0.3
26.0	13.2	10.0	0.9	1.0	0.3	1.0	4.5	0.4	−0.7
24.0	13.2	9.7	0.7	0.8	0.8	0.0	2.6	0.3	−0.2
23.0	13.2	9.6	1.1	0.8	0.8	0.0	2.7	0.6	0.1
22.0	13.1	9.6	1.5	0.8	0.8	0.0	2.8	0.4	0.0
21.0	13.3	9.6	1.3	0.0	0.8	0.0	1.2	−1.1	−1.5
19.0	13.0	9.3	1.1	0.0	0.5	1.0	1.1	1.9	0.0
18.0	13.0	9.0	2.3	0.0	0.5	0.0	1.3	0.2	0.4
17.0	13.0	9.0	1.4	0.0	0.5	0.0	1.3	−1.2	−1.4
17.0	12.9	9.0	1.0	0.0	0.5	0.0	1.0	0.9	0.1
16.0	12.8	9.1	1.1	0.0	0.5	0.0	1.1	1.0	0.1
13.0	12.7	9.1	0.8	0.0	0.5	1.0	1.2	0.0	−0.1
12.0	12.8	9.1	1.4	0.0	0.5	0.0	1.3	0.7	0.7
11.0	12.8	9.3	1.4	0.0	0.5	0.0	1.3	0.5	0.6
11.0	12.8	9.6	1.3	0.0	0.5	0.0	1.2	0.4	0.4
10.0	13.0	9.8	1.1	0.0	0.5	0.0	1.2	0.1	0.0
9.0	12.8	9.5	0.7	0.0	0.5	1.0	1.2	0.5	−0.2
9.0	13.0	9.2	1.1	0.0	0.5	0.0	1.3	0.4	0.4
12.0	13.0	9.0	0.9	0.5	0.5	0.0	1.3	0.6	0.2
13.0	13.4	8.8	0.9	0.0	0.3	0.0	1.3	−0.3	0.0
15.0	13.3	8.7	0.7	0.0	0.3	0.0	1.2	0.4	0.4
								$\sum_{n=1}^{30} \|y-y'\| = 19.8$	$\sum_{n=1}^{30} \|y-y'\| = 12.8$

The results of the tuning (more then ten thousand steps) of the membership functions of the antecedent parts are shown in Fig. 5. The tuned values of the consequent parts are

$$w_1 = 2.45,\ w_2 = -4.39, w_3 = 8.80,\ w_4 = -4.07, w_5 = 2.50,$$
$$w_6 = 11.02, w_7 = 5.68,\ w_8 = 0.85,\ w_9 = 3.90,\ w_{10} = 0.00$$

After tuning, the sum of the absolute values of the errors became equal to 12.8 (Table 1). The values of the partial derivatives in tuning rules were near zero but not still equal to zero. So the procedure can be still continued and errors can be reduced.

CONCLUSION

We propose a self-tuning method for the fuzzy membership functions from class C^0. Membership functions from class C^l, $(l \geq 1)$ can also be tuned in the same way. The algorithm uses the expert's defined input-output data to determine the values of the parameters of the membership functions. After obtaining the appropriate set of rules

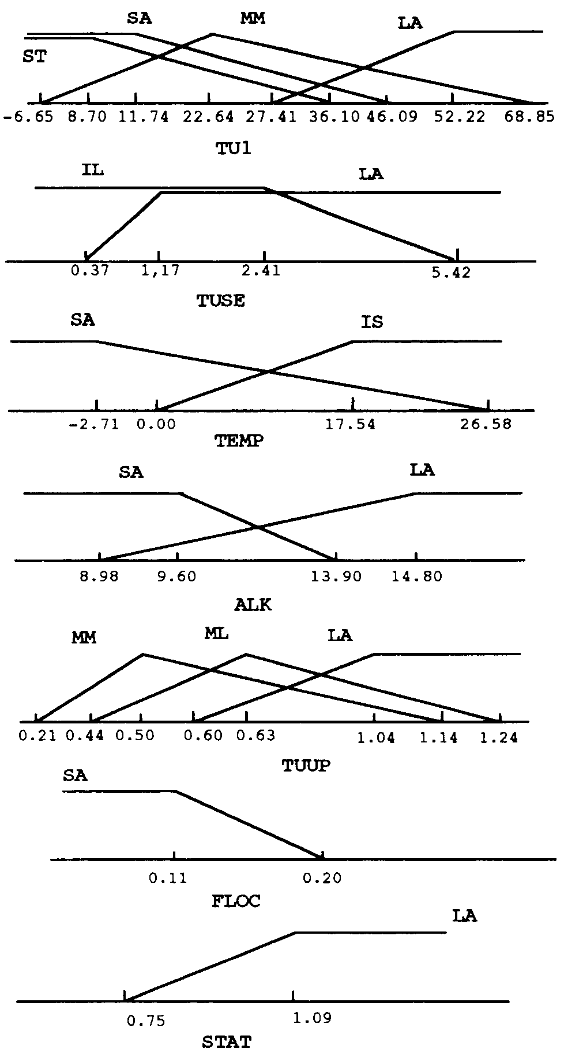

Fig. 4 Membership functions of the antecedent parts in [7].

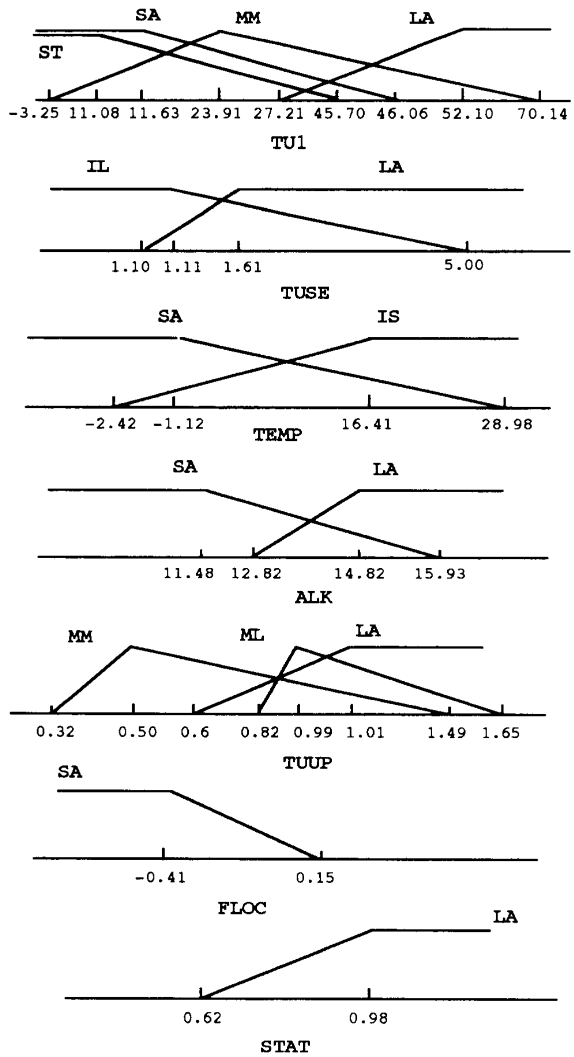

Fig. 5 Membership functions after tuning.

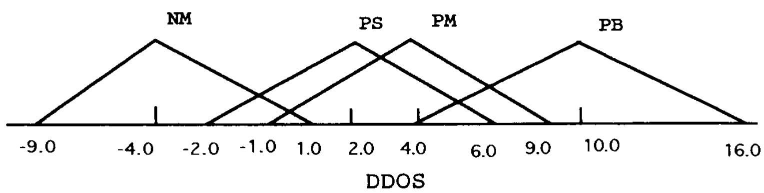

Fig. 6 Membership functions of the consequent parts.

and training data this method can be useful for tuning membership functions. Tuned membership functions considerably improve the performance. This research extends and enhances tuning algorithm developed by Ichihashi and Nomura. The water purification control system is used to demonstrate the algorithm.

REFERENCES

1) Kong So. and Kosko B., "Adaptive Fuzzy Systems for Backing up a Truck-and-Trailer" IEEE

transactions on neural networks, Vol. **3**, No. 2, 1992.

2) Ollero A. and Garcia-Cerezo A., "Direct Digital Control, Auto-Tuning and Supervision using fuzzy Logic" Fuzzy sets and Systems 30, Nort-Holland 1989.

3) Mallampati D. and Shenoi So., "Self-Organizing Fuzzy Logic Control" Knowledge-Based Systems and Neural Networks, Elsevier Science Publishing, 1991.

4) Brown M. and Harris C., "A Nonlinear Adaptive Controller: A Comparison between Fuzzy Logic Control and Neurocontrol" IMA Journal of Mathematical Control and Information, 1991.

5) Chang C., Abdelnour G. and Cheung Y., "Adaptive Mapping Factors for Tuning Fuzzy Controler" Knowledge-Based Systems and Neural Networks, Elsevier Science Publishing, 1991.

6) Walter H.B., Mulholland R.J. and Sofer So.So., "Design of a Self-Tuning Rule Based Controller for a Gasoline Refinery Catalytic Reformer" IEEE transactions on automatic control, Vol. **35**. No. 2. 1990.

7) 汎用ファジィコントローラ FRUITAX のマニュアル. 富士電機, 1985.

8) 市橋, 田中：PID とFuzzy のハイブリット型コントローラー. 第 4 回ファジィシステムシンポシウム, 1998.

9) 市橋秀友：システム制御情報チュートリアル講座 '89制御工学へのガイドらん. 最新の理論のプロフィルと適用の実際, 1988.

10) 市橋秀友：階層的ファジィモデルによる誤差逆伝播学習. 第 6 回ファジィシステムシンポシウム, 1990.

11) 野村, 林, 若見：最急降下法によるファジィ推論の自動チューニングと障害物回避への用. 第 6 回ファジィシステムシンポシウム, 1990.

12) 野村, 荒木, 林, 若見：デルタルールによる学習型ファジィ推論. 日本機械学会 FAN シンポシウム講演論文集, 1991.

13) Donald A. Pierre., "Optimization Theory with applications", Dover Publications, 1990.

14) King P. and Mamdani E., "The Application of Fuzzy Control Systems to Industrial Processes" Automatica **13**, 1987.