The Appearence Order and Structure of Oscillation mode of OREGONATOR Exerted by an External Periodic Oscillation Force.

Takahiro YAMAGUCHI and Kuninosuke IMAEDA

Department of Electronic Engineering, Okayama University of Science, Ridaicho, 1-1 Okayama, 700, Japan.

(Received September 30, 1987)

Abstract

We have investigated the response of a nonlinear dynamical system : OREGONATOR expressed by a set of nonlinear ordinary differential equations under the infulence of an external periodic force.

The OREGONATOR simulates the Belousov-Zhabotinskii reaction.

We have found that this system exhibits a band structure of oscillation modes.

1 Introduction

Recently many experimental studies have been reported on the Belousov-Zhabotinskii (B-Z) reaction in an isothermal, open, constant stirred tank reactor (CSTR)[1-3]

The (B-Z) reaction which is taking place in CSTR exhibits complex phenomena compared with that in the closed system such as multiperiodic oscillations and chaos phenomena. Hudson et al. have investigated the complex phenomena such as multiple peak periodic oscillations in the concentration of bromide ions they have shown that multiperiodic oscillations lead to chaos. Field, Körös and Noyes[4] have shown that the oscillation can be explained by so called "FKN mechanisum".

Field and Noyes invented a kinetic model "Oregonator" which is based on the five steps of constituent reactions. In this paper , we use the OREGONATOR used by Tyson[5].

Tyson presented a scheme of analysis of the differential equations of the

OREGONATOR giving a reasonable agreement between model calculations and observed oscillations. Thus, the OREGONATOR has proved to be a realistic model which is based on the kinetics of the BZ reaction. In recent papers[8,12], the response of the OREGONATOR under the influence of an external force has been studied by adding the third component an external periodic force. By extending the study, we investigate the property of the nonlinear dynamical system: the OREGONATOR under the influence of an external periodic force through the change of responce in the oscillation modes.

In order to attain this purpose, we investigate the response in the oscillation modes of OREGONATOR by adding to the second component (case 1) or third component (case 2) an external periodic force. In the case 1, in the parameter space of the amplitude and frequency of the external periodic oscillation force, a band structure of oscillation modes appears. In the case 2, the multiple periodic oscillation modes appear in a successive order in the multiplicity, except for a region of small $A\rho$.

2 Foundamental Equations and calculations

To study the behaviour of a nonlinear system, the OREGONATOR expresed by a set of three ordinary nonlinear differential equations exerted by an external periodic force, we have adopted the Oregonator for the BZ reaction as adopted by Tyson with an addition of an external periodic driving force.

The set of nonlinear differential equations is as follows

$$\varepsilon d\xi/d\tau = \xi + \eta - q\xi^2 - \xi\eta$$
$$d\eta/d\tau = -\eta + 2hp - \xi\eta + A\eta COS(\omega_\eta \tau)$$
$$pd\rho/d\tau = (\xi - \rho) + A\rho COS(\omega_\rho \tau)$$

Where we have used the same notation as Tyson's: ξ, ρ are variable, ε, p, q, h are constants and are chosen the same values as those used by Field et al: 0. 03, 2.0, 0.006, 0.75, respectively and A_i COS $(\omega\tau)$ are the external force. In the simulation calculation, we take the external parameters as $A\eta$, $A\rho$ and ω_η.

We have studied the response in η by fixing amplitude A_i ($i=\eta$, ρ) being constants and frequency ω_i ($i=\eta$, ρ) being varied.

3 Result

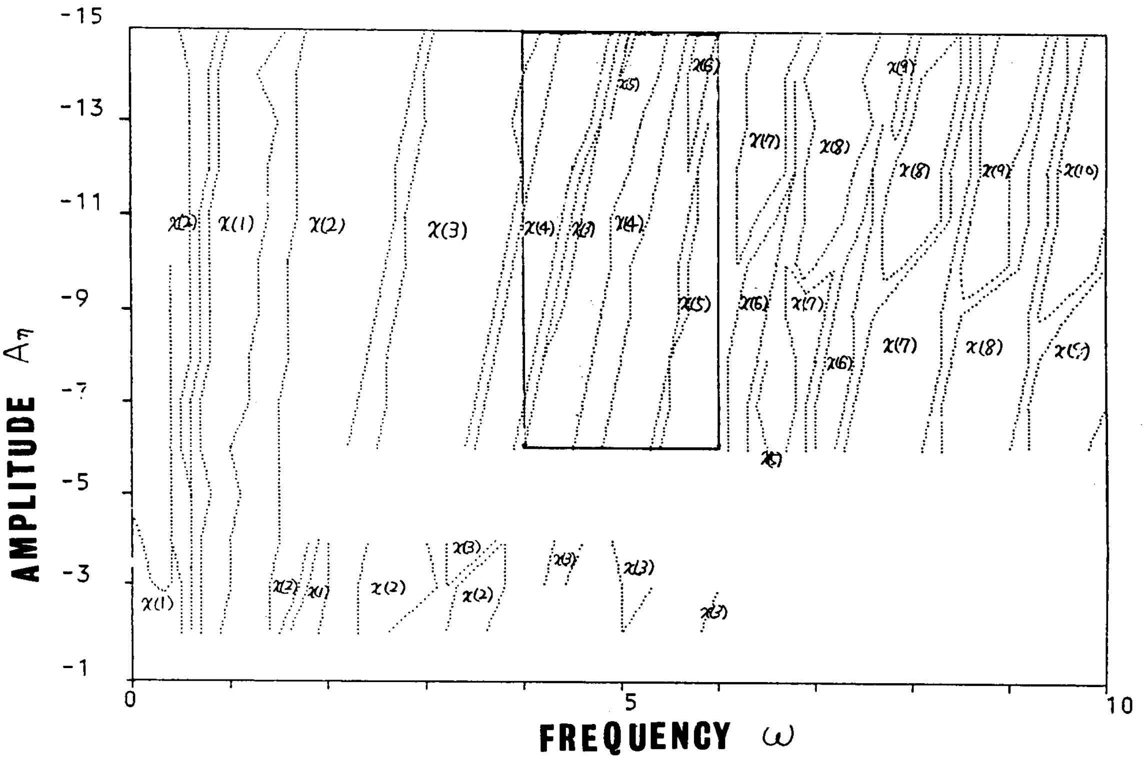


Fig 1 Mode diagrams in the parameter space of the amplitude A_η and frequency ω_η when the external periodic force $A_\eta COS(\omega\tau)$ is exerted to the second component of OREGONATOR.

χ (m) : represents a foundamental peak oscillation (m-PO). A dotted line (·········) represents the boundary of a region where an m. PO mode may possibly appear.

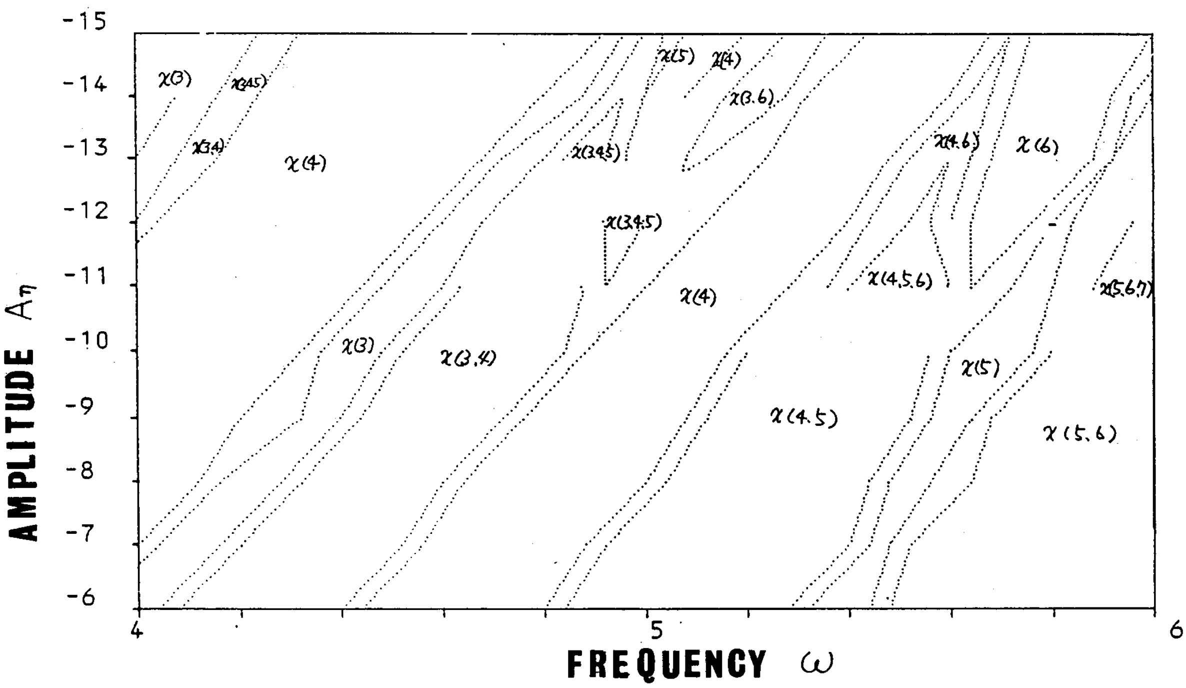


Fig 2 An enlanged figure of the region in the parameter space surrounding by real lines in Fig 1.

χ (m, m+1) : mixed mode of m-.PO and m+1-. PO.

χ (m, m+1, m+2) : a mixed mode of m-PO, m+1-. PO and m+2-.PO

Case 1:

We illustrate the result for the case 1: adding to the second component of

OREGONATOR with an external periodic force and ω_η being varied

In Fig 1, the foundamental oscillation mode χ (m)=1,2,3,·········does not appear in the sequential order. In this figure, we find that the multiple oscillation mode depends sensitively on the amplitude $A\eta$ of the external periodic force. Furthermore, in order to show the·appearence of the oscillation mode in detail, enlarged the area surrounded by the real lines in Fig 1.

Fig 2, the multiple peak oscillation mode χ (3,4,5) appears in the neighborhood of $A\eta=-13$, $\omega=4.9$.

χ (3,4,5) is the mixture of osillation modes of three oscillation modes χ (3), χ (4), χ (5).

The region where χ (3,4,5) appear, is given in the next section.

Case 2:

Now, we illustrate the case 2: adding to the third component of OREGONATOR with an external periodic force and ω_ρ being varied.

In Fig 4 fundamental peak oscillation modes χ (m) appear in sequential order, except for a region of small $A\rho$ in the parameter space of ($A\rho$, ω_ρ).

In this case, at a region between the two adjacent fundamental oscillation

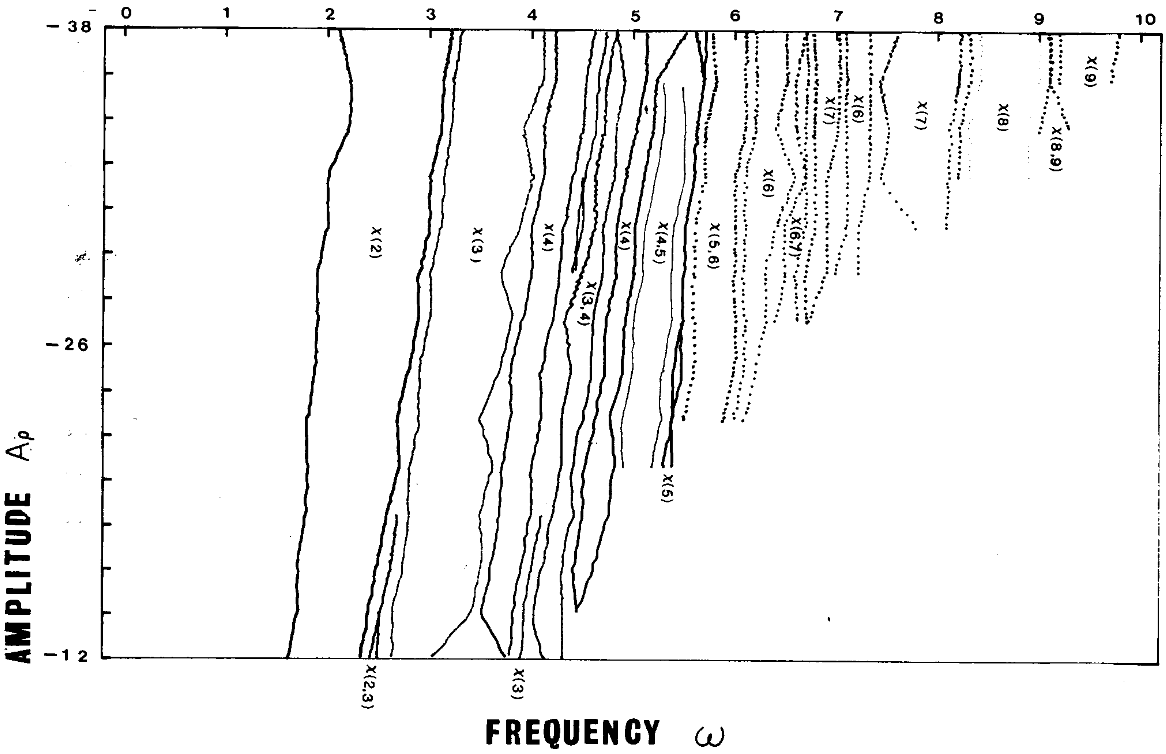


Fig 3 The phase diagram in the parameter space for oscillation modes of the OREGONATOR exerted by an external periodic force $A\rho$ COS (ω_ρ τ) of amplitude $A\rho$ and frequency ω for the case 2: added to the third component of OREGONATOR with an external periodic force.

Fig 4(a)

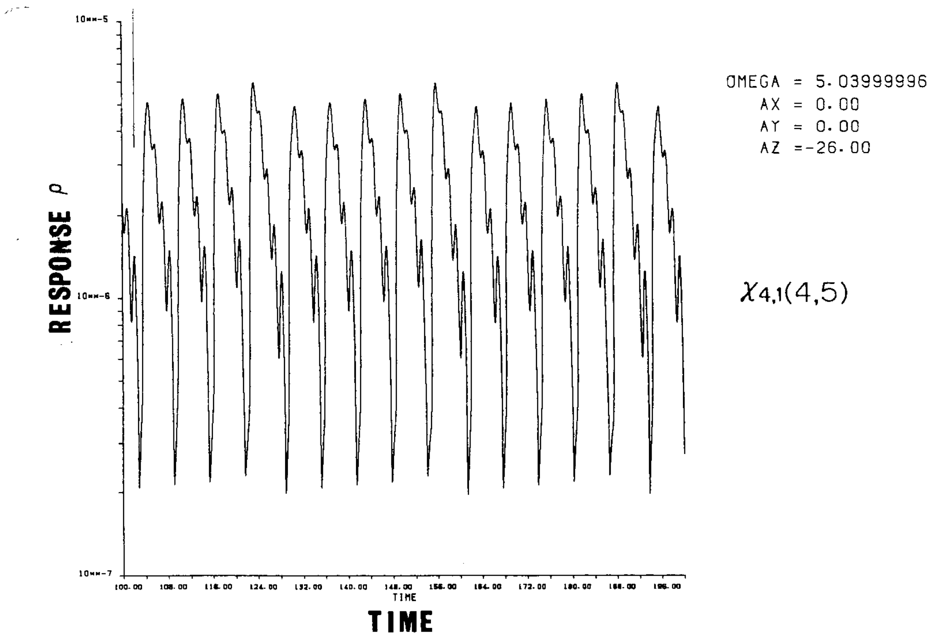


Fig 4(b)

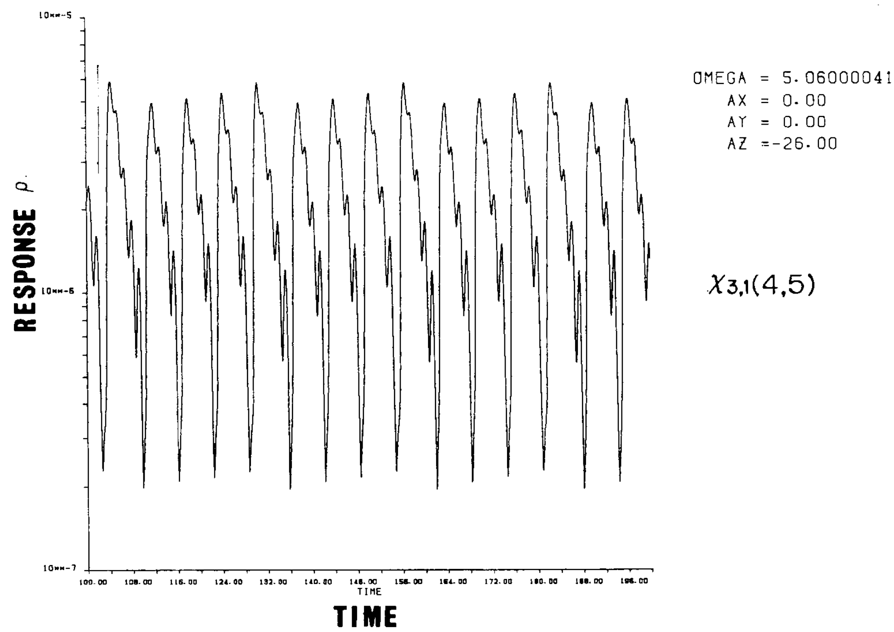


Fig 4 From Fig 4-a through Fig 4-f represent the state of change of modes from χ (4) to χ (5) for a region where a foundamental peak oscillation modes appear in a successive order in the multiplicity in Fig 3. $\chi_{p,q}$ (m,m+1) represents a multiple oscillation mode consisting of P m-PO and q m+PO,

modes, a multiple peak oscillation mode χ (m, m+1) appears, where χ (m, m+1) is a mixed mode of m-PO and m+1-PO. For example, states in the region between χ (4) and χ (5) modes are illustrated in Fig 4-a through fig 4-f.

Fig 4(c)

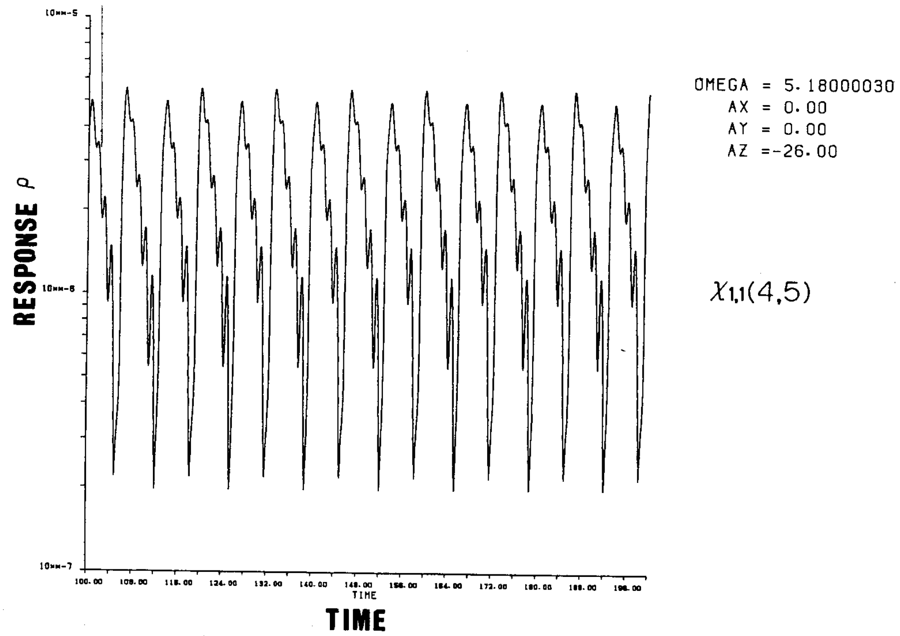
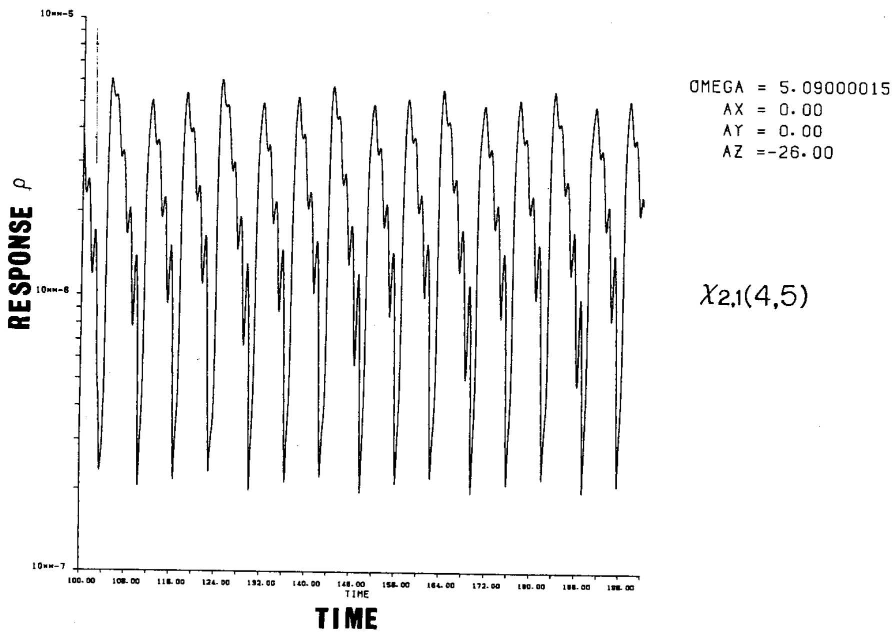
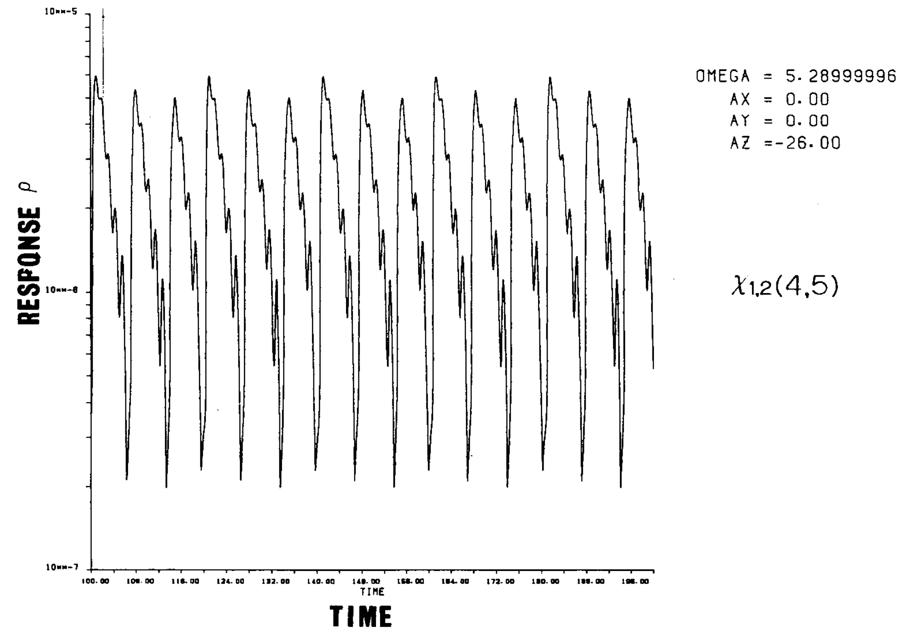


Fig 4(d)

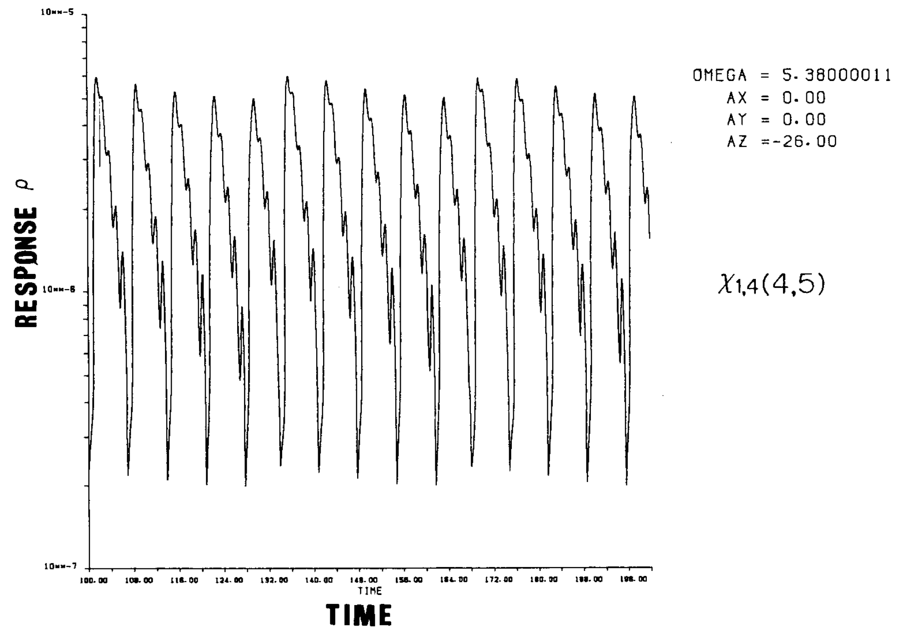


The $\chi_{p,q}$ (m, m+1) denotes a multiple fundamental oscillation mode χ (m, m+1) and it represents an oscillation mode consisting of number P. times of m-PO and the number q. time of m+1-PO.

Fig 4(e)



Fig(f)



The more the mixing ratio p/p+q becomes large the more a mode approaches to the χ (m) and the less the mixing ratio becomes, the more a mode approaches to χ (m+1).

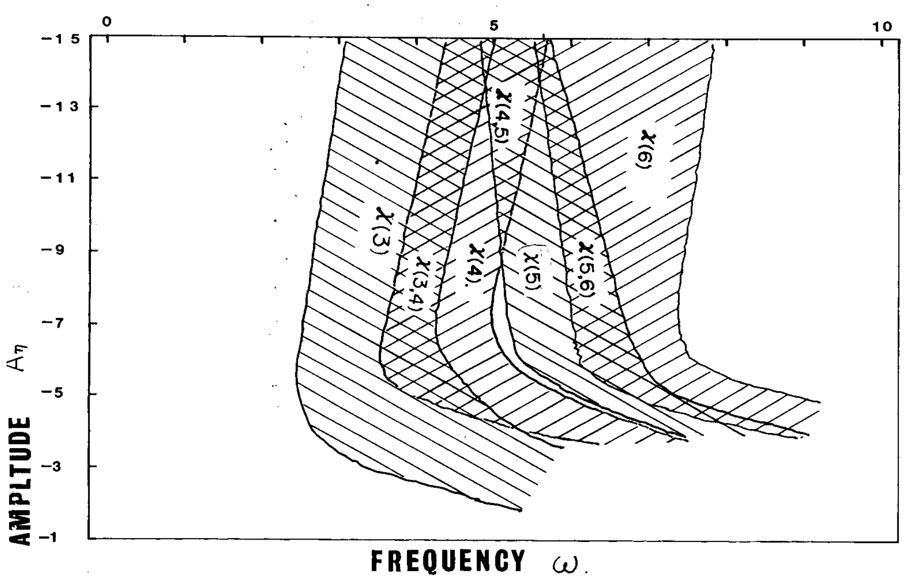


Fig 5 The figure represents regions where the modes: χ (3), χ (4) χ (5), χ (6), χ (3,4), χ (4,), χ (5,6) may appear for the case when an external periodic force is applied on the second component of the OREGONATOR.

4 Conclusion and discussion

As described in a previous section 3, in Fig 2, a multiple peak oscillation mode χ (3,4,5) is observed. In order to explain the appearance of the multiple peak oscillation mode χ (3,4,5), we introduce a concept of a band structure for a mode in the parameter space (A_η, ω_η). Here, a band is defined as the region where a m-.peak oscillation mode can appear. The regions of bands χ (3), χ (4), χ (5), χ (6) etc. and those of double mixed bands χ (2,3), χ (3,4), etc. and where three band of χ (3). χ (4), and χ(5) supperposed are shown in Fig 3.

The multiple peak oscillation mode χ (3,4,5) appears, in the region where three bands of χ (3), χ (4),and χ (5) superposed.

The existence of the χ (3,4,5) mode is explained by introducing the concept of bands.

Furthermore, we are going to solve the band structure of the oscillation mode theoretically by investgating the oscillation mode in a region where the band overlapped.

Acknowledgement

We wish to thank Mr. Y. Yamamoto for the valuable discussion and Mr. A. Kurokawa for the contribution to the calculation and the introduction of the band concept.

6 Reference.

1) J.L. Hudson, M. Hart and D. Marinko: J. Chem. Phys. **71, 1601 (1979)**

2) C. Vidal, J.C.Roux, S. Bacheiart, and A. Rossi, C. R. Acad. : Paris. **289, 73 (1979)**

3) J. C. Roux: Physica. **7D, 57 (1983)**

4) R.J. Field, E. Körös and R. M. Noyes: J. Am. Chem. Soc. **94,8649 (1972)**

5) J. J. Tyson: Belousov-Zhabotinskii Reaction. "Lecture note in biomathematics." Vol.**10**, Springer-Verlag, Berlin, **1976**.

6) K. Tomita and I. Tsuda: Prog. Theor. Phys. **64, 1138 (1980)**

7) K. Showalter, R. M. Noyes, and K. Bar-Eli, J. Chem. Phys, **69, 2514 (1978)**

8) K. Imaeda and Tei: Journ. Phys. Soc. Japan. **55,743 (1986)**

9) J. L. Hudson and J. C. Mankin: "Stochastic Phenomena and Chaos Behavior in Complex system" Ed. P. Schuster, Springer Verlag. **p98 (1984).**

10) T. Koyama and Y. Aizawa: Prog. Theor. Phys. **70, 1002 (1983)**

11) H. Mori, H. Okamoto, and M. Ogasawara: Prog Theor. Phys. **499, 71 (1984).**

12) K. Imaeda, Y. Yamamoto, and T. Ymaguchi: Journ. Phys. Soc. Japan 56, 3832 (1987).